Internet Appendix for "Networks in Production: Asset Pricing Implications"

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This Internet Appendix contains additional results and robustness tests to the paper. In Sections I, II, and III, I provide a detailed derivation of the model. In Section IV, I discuss a version of the model that features a competitive labor market. In Section V, I set up a version of the model with CES investment aggregator and solve a first-order approximation of the model around the unit-elastic case. In Section VI, I discuss a calibration of the model. In Section VII, I discuss the data construction. In Section VIII, I conduct a truncation of analysis, and in Section IX I report additional tables and figures.

I. Solving Firms' and Household's Optimization Problems

In this appendix, I solve a more general specification of the model in which the investment aggregator function has constant elasticity of substitution given by ν. Firms i's optimization problem is given by

$$D_{i,t} = \max_{\{y_{ij,t}\}_j, I_{i,t}} P_{i,t}\varepsilon_{i,t}I_{i,t}^{\eta} - \sum_{j=1}^{n} P_{j,t}y_{ij,t} \qquad \text{subject to} \qquad I_{i,t} = \left[\sum_{j=1}^{n} w_{ij}y_{ij,t}^{1-1/\nu}\right]^{\frac{1}{1-1/\nu}},$$

where ν is the elasticity of substitution between two distinct inputs and $\eta \in (0,1)$ is the returns to scale. Let $\mu_{i,t}$ be the Lagrange multiplier on the constraint. The first-order conditions are

$$y_{ij,t} : -P_{j,t} + \tilde{\mu}_{i,t}I_{i,t}^{1/\nu}y_{ij,t}^{-1/\nu}w_{ij} = 0 \implies y_{ij,t} = \mu_{i,t}^{\nu}\frac{w_{ij}^{\nu}I_{i,t}}{P_{j,t}^{\nu}},$$

and

$$I_{i,t} : -\mu_{i,t} + \eta P_{i,t}\varepsilon_{i,t}I_{i,t}^{\eta-1} = 0 \implies I_{i,t} = \left(\frac{\eta P_{i,t}\varepsilon_{i,t}}{\mu_{i,t}}\right)^{\frac{1}{1-\eta}}.$$

The remaining Karush–Kuhn–Tucker condition is the investment constraint itself. Using the optimality conditions, we may simplify the Lagrange multiplier as follows:

$$I_{i,t} = \left[\sum_{j=1}^{n} w_{ij}y_{ij,t}^{1-1/\nu}\right]^{\frac{1}{1-1/\nu}} = \left[\sum_{j=1}^{n} w_{ij}\left((\mu_{i,t})^{\nu}\frac{w_{ij}^{\nu}I_{i,t}}{P_{j,t}^{\nu}}\right)^{1-1/\nu}\right]^{\frac{1}{1-1/\nu}},$$

which implies

$$1 = (\mu_{i,t})^{\nu}\left[\sum_{j=1}^{n} w_{ij}\left(\frac{w_{ij}^{\nu}}{P_{j,t}^{\nu}}\right)^{1-1/\nu}\right]^{\frac{1}{1-1/\nu}}.$$

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Therefore,

$$\mu_{i,t} = \left[\sum_{j=1}^{n} w_{ij} \left(\frac{w_{ij}^{\nu}}{P_{j,t}^{\nu}} \right)^{1-1/\nu} \right]^{-\frac{1/\nu}{1-1/\nu}} = \left[\sum_{j=1}^{n} w_{ij}^{\nu} P_{j,t}^{1-\nu} \right]^{\frac{1}{1-\nu}}.$$

For $\nu = 1$, we have $\mu_{i,t} = \frac{\prod_{j=1}^{n} P_{j,t}^{w_{ij}}}{\prod_{j=1}^{n} w_{ij}^{w_{ij}}}$. Hence, $\mu_{i,t}$ is a network-weighted average of spot market prices, which according to the investment first-order condition is equal to the market value of an extra unit of investment. We have, therefore, derived equations (8), (9), and (10) in the paper, which fully specify firms' optimality conditions.

For the household's optimization problem, let λ_t be the Lagrange multiplier for the period t budget constraint of equation (5) in the paper. Then the first-order condition for $c_{i,1}$ is given by

$$\mathcal{C}_t^{-\gamma} \frac{\partial \mathcal{C}_t}{\partial c_{i,t}} = P_{i,t} \lambda_t.$$

This implies that

$$\frac{\frac{\partial \mathcal{C}_t}{\partial c_{i,t}}}{\frac{\partial \mathcal{C}_t}{\partial c_{j,t}}} = \frac{P_{i,t}}{P_{j,t}},$$

which represents the intra-period consumption allocation. For a Cobb-Douglas aggregator, this consumption allocation rule becomes

$$c_{i,t} = \alpha_i \frac{\sum_{i=1}^{n} D_{i,t}}{P_{i,t}}.$$

The intertemporal consumption allocation rule from the first order condition is given by:

$$\mathbb{E}_t \left[\underbrace{\beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} \right)^{-\rho} \frac{\frac{\partial \mathcal{C}_{t+1}}{\partial c_{1,t+1}} / P_{1,t+1}}{\frac{\partial \mathcal{C}_t}{\partial c_{1,t}} / P_{1,t}}}_{\equiv M_{t+1}} \underbrace{\frac{V_{i,t+1}}{V_{i,t} - D_{i,t}}}_{\equiv R_{i,t+1}} \right] = 1.$$

We thus have equation (12) in the paper.

II. Price Normalization

A. *Simplifying the Stochastic Discount Factor*

Prices are normalized to simplify the SDF expression. Specifically, prices are normalized such that the marginal aggregator term is set to one:

$$\frac{\partial \mathcal{C}_t}{\partial c_{i,t}} / P_{i,t} = 1 \ \ \forall i, t.$$

It turns out that the necessary price normalization has an intuitive interpretation: it makes the risk-free rate of return equal to the return on a claim to the aggregator utility. To find the necessary price normalization, write marginal aggregator in the SDF in terms of spot market prices:

$$1 = \frac{\partial \mathcal{C}_t}{\partial c_{i,t}} / P_{i,t} = \frac{\alpha_i \mathcal{C}_t}{P_{i,t} c_{i,t}} = \frac{\alpha_i}{P_{i,t}} \prod_{j=1}^{n} \left(\frac{c_{j,t}}{c_{i,t}} \right)^{\alpha_j} = \frac{\alpha_i}{P_{i,t}} \prod_{j=1}^{n} \left(\frac{\alpha_j / P_{j,t}}{\alpha_i / P_{i,t}} \right)^{\alpha_j} = \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}} \right)^{\alpha_j}.$$

Therefore, prices are normalized such that

$$\prod_{j=1}^{n} P_{j,t}^{\alpha_j} = \prod_{j=1}^{n} \alpha_j^{\alpha_j}.$$

We thus have equation (13) in the paper.

B. *Price Normalization and Returns*

The risk-free rate of return is given by

$$R_{rf,t} = \frac{1}{\mathbb{E}_t \left[M_{t+1} \right]}.$$

The risk-free rate of return is sensitive to the price normalization chosen. For example, if good 1 is defined as the numeraire, then the risk-free rate of return is the return on a claim to one unit of good 1. Since all relative prices may change next period, this claim is still risky as one unit of good 1 may buy different units of other goods. Hence, when the household buys such a claim, its payoff (in terms of aggregator utility) is random, because he will substitute consumption goods in order to maximize utility.

To avoid this issue, I define the risk-free rate as the return on a riskless bundle, specifically, a bundle that cancels this substitution effect keeping the aggregator utility constant. Hence, I define the risk-free return as the return on a claim to the aggregator utility (see definition below). This bundle has to satisfy the intra-period first-order condition of the representative agent and its price has to be normalized to one.

DEFINITION IA 1 (Riskless Claim): *The riskless claim is a claim to the following bundle:*

$$\left(\frac{\alpha_1}{P_{1,t}}, \dots, \frac{\alpha_n}{P_{n,t}} \right).$$

The price of such a claim is one because $\sum_{j=1}^{n} \alpha_j = 1$. The riskless claim satisfies the intra-period first-order condition and is a claim to the consumption aggregator: buying κ units of such a claim yields the following consumption aggregator amount:

$$\mathcal{C}_t = \kappa \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}} \right)^{\alpha_j} = \kappa.$$

C. *Consumption Expenditure and Utility Aggregator*

Another useful property of the price normalization chosen is that consumption expenditure and the utility aggregator are the same in equilibrium. Evaluating the consumption aggregator in equilibrium at the normalized prices yields

$$\mathcal{C}\left(c_t\right) = \prod_{j=1}^{n} c_{j,t}^{\alpha_j} = \prod_{j=1}^{n} \left(\frac{\alpha_j \omega_t}{P_{j,t}}\right)^{\alpha_j} = \omega_t \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}}\right)^{\alpha_j} = \omega_t.$$

III. Closed-Form Expressions: Derivations

In this section, I derive equilibrium aggregate consumption growth as a function of residual TFP, network concentration, and network sparsity. First, I derive the equilibrium output shares. Next, I characterize the system of equations that fully specifies the equilibrium. Finally, I derive the consumption growth process and the network factors in closed form.

A. *Output Share*

Starting form the market-clearing condition for good i, we have

$$c_{i,t} + \sum_j y_{ji,t} = Y_{i,t},$$

$$c_{i,t} + \sum_j \mu_{i,t}^{\nu} \frac{w_{ji,t}^{\nu} I_{j,t}}{P_{i,t}^{\nu}} = Y_{i,t},$$

$$P_{i,t} c_{i,t} + \sum_j \mu_j^{\nu} w_{ji,t}^{\nu} P_{i,t}^{1-\nu} I_{j,t} = P_{i,t} Y_{i,t},$$

$$\alpha_i (1-\eta) z_t + \eta \sum_j w_{ji,t}^{\nu} \left(\frac{P_{i,t}}{\mu_j}\right)^{1-\nu} P_{j,t} Y_{j,t} = P_{i,t} Y_{i,t},$$

$$\alpha_i (1-\eta) z_t + \eta \sum_j \frac{w_{ji,t}^{\nu} P_{i,t}^{1-\nu}}{\sum_s w_{js}^{\nu} P_s^{1-\nu}} P_{j,t} Y_{j,t} = P_{i,t} Y_{i,t},$$

and

$$(1-\eta) \alpha_i z_t + \eta \sum_j P_{j,t} Y_{j,t} \tilde{w}_{ji,t} = P_{i,t} Y_{i,t},$$

where $z_t = \sum_i P_{i,t} Y_{i,t}$ is the total output of the economy in period t and $\tilde{w}_{ji,t} = \frac{w_{ji,t}^{\nu} P_{i,t}^{1-\nu}}{\sum_s w_{js}^{\nu} P_s^{1-\nu}}$.

In matrix notation, the above system becomes

$$(1-\eta) \alpha z_t + \eta \tilde{W}_t' \overrightarrow{PY}_t \quad = \quad \overrightarrow{PY}_t,$$

where $\overrightarrow{PY}_t$ is a column vector of sectors' output, $\tilde{W}_t$ is a $n \times n$ matrix whose (i,j) entry is $\tilde{w}_{ji,t}$, and α is a column vector of preference weights. We can solve this system and express the output of each firm as a fraction of the total output,

$$\overrightarrow{PY}_t = \underbrace{(1-\eta) \left[\mathbb{I} - \eta \tilde{W}_t'\right]^{-1} \alpha}_{\equiv \delta_t} z = \delta_t z_t,$$

where δ_t is a $n \times 1$ vector of output shares for each sector/firm.

When $\nu = 1$, the output shares are completely determined by the network and household preferences:

$$\delta_t = (1-\eta)\left[\mathbb{I} - \eta W_t'\right]^{-1}\alpha.$$

We thus have equation (15) in the paper.

B. *Equilibrium Conditions*

The SDF, however, depends on changes in the aggregate consumption expenditure, that is, $\log\left(\frac{\omega_{t+1}}{\omega_t}\right)$. Consumption expenditure is proportional to total output,

$$\omega_t = \sum_j c_{j,t} P_{j,t} = \sum_j D_{j,t} = (1-\eta)\sum_j P_{j,t} Y_{j,t} = (1-\eta) z_t.$$

This implies that total output growth is equal to total consumption expenditure growth:

$$\log\left(\frac{\omega_{t+1}}{\omega_t}\right) = \log\left(\frac{z_{t+1}}{z_t}\right).$$

Using the first-order condition of firm i, we can derive equation (16) in the paper as follows:

$$\begin{aligned}
\delta_{i,t} z_t &= P_{i,t} Y_{i,t}, \\
\delta_{i,t} z_t &= \frac{\mu_{i,t} I_{i,t}}{\eta}, \\
\delta_{i,t} z_t &= \frac{\mu_{i,t}\left(\frac{\eta P_{i,t}\varepsilon_{i,t}}{\mu_{i,t}}\right)^{\frac{1}{1-\eta}}}{\eta}, \\
\delta_{i,t} z_t &= \mu_{i,t}^{-\frac{\eta}{1-\eta}} P_{i,t}^{\frac{1}{1-\eta}} \varepsilon_{i,t}^{\frac{1}{1-\eta}} \eta^{\frac{\eta}{1-\eta}},
\end{aligned}$$

and

$$(\delta_{i,t} z_t)^{1-\eta} = \mu_{i,t}^{-\eta} P_{i,t} \varepsilon_{i,t} \eta^{\eta}.$$

C. *Output Growth: Cobb-Douglas Case*

Assuming that $\nu = 1$, the Lagrange multiplier becomes

$$\mu_{i,t} = \prod_j \frac{P_{j,t}^{w_{ij,t}}}{w_{ij,t}^{w_{ij,t}}}.$$

Substituting the multiplier into equation (16), we have

$$\begin{aligned}
(\delta_{i,t} z_t)^{1-\eta} &= \mu_{i,t}^{-\eta} P_{i,t} \varepsilon_{i,t} \eta^{\eta}, \\
(1-\eta)\log\delta_{i,t} + (1-\eta)\log z_t &= -\eta\log\mu_{i,t} + \log P_{i,t} + \log\varepsilon_{i,t} + \eta\log\eta,
\end{aligned}$$

and

$$\begin{aligned}
(1-\eta)\log\delta_{i,t} + (1-\eta)\log z_t = &-\eta\sum_j w_{ij,t}\log P_{j,t} + \eta\sum_j w_{ij,t}\log w_{ij,t} \\
&+ \log P_{i,t} + \log\varepsilon_{i,t} + \eta\log\eta.
\end{aligned}$$

Writing the above system in matrix notation and using the price normalization yields

$$(1-\eta)\log\delta_t + (1-\eta)\mathbf{1}\log z_t = -\eta W\log P_t + \eta\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t + \log P_t + \log\varepsilon_t + \mathbf{1}\eta\log\eta,$$

which is simplified to

$$\log P_t = [\mathrm{I}-\eta W]^{-1}\left((1-\eta)\log\delta_t + (1-\eta)\mathbf{1}\log z_t - \eta\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log\varepsilon_t - \mathbf{1}\eta\log\eta\right),$$

$$\alpha'\log P_t = \alpha'[\mathrm{I}-\eta W]^{-1}\left((1-\eta)\log\delta_t + (1-\eta)\log z_t - \eta\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log\varepsilon_t - \eta\log\eta\right),$$

$$\alpha'\log\alpha = \delta_t'\left(\log\delta_t + \mathbf{1}\log z_t - \frac{\eta}{1-\eta}\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta}\log\varepsilon_t - \mathbf{1}\frac{\eta}{1-\eta}\log\eta\right),$$

and

$$\alpha'\log\alpha = \log z_t - \frac{\eta}{1-\eta}\log\eta + \delta_t'\left(\log\delta_t - \frac{\eta}{1-\eta}\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta}\log\varepsilon_t\right),$$

where $\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t$ is a $n\times 1$ vector with the i^{th} element being $\mathcal{N}^{\mathcal{S}}_{i,t} = \sum_j w_{ij,t}\log w_{ij,t}$ and δ_t is a $n\times 1$ vector with output shares as in equation (15) in the paper.

Thus, output in equilibrium is given by

$$\log z_t = \alpha'\log\alpha + \frac{\eta}{1-\eta}\log\eta - \delta_t'\left(\log\delta_t - \frac{\eta}{1-\eta}\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta}\log\varepsilon_t\right).$$

Taking a one period difference, equilibrium output growth is as stated in Theorem 1:

$$\log z_{t+1} - \log z_t = -\left(\mathcal{N}^{\mathcal{C}}_{t+1} - \mathcal{N}^{\mathcal{C}}_t\right) + \frac{\eta}{1-\eta}\left(\mathcal{N}^{\mathcal{S}}_{t+1} - \mathcal{N}^{\mathcal{S}}_t\right) + \frac{1}{1-\eta}\left(e_{t+1} - e_t\right).$$

Furthermore, the equilibrium spot market prices are given by

$$\begin{aligned}\log P_t &= [\mathrm{I}-\eta W_t]^{-1}\left[(1-\eta)\log\delta_t + (1-\eta)\mathbf{1}\log z_t - \eta\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log\varepsilon_t - \mathbf{1}\eta\log\eta\right] \\ &= \log z_t - \frac{\eta}{1-\eta}\log\eta + (1-\eta)[\mathrm{I}-\eta W_t]^{-1}\left[\log\delta_t - \eta\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log\varepsilon_t\right].\end{aligned}$$

IV. Model with Competitive Labor Market

In this section, I consider a competitive labor market in addition to the framework discussed in Section I of the main paper. This extension features diminishing returns to input investments, and the production function is given by

$$Y_{i,t} = \varepsilon_{i,t} I^{\eta}_{i,t} L^{\psi}_{i,t},$$

where $\psi \geq 0$ and $L_{i,t}$ is an additional production factor (e.g., labor or land) used as an input. In addition, I assume $\psi + \eta \leq 1$ to rule out increasing returns to input investment.

The benchmark model is equivalent to assuming $L_{i,t} = 1$ for each sector i and period t (see equation (2) in the paper). In this section, I relax this assumption and model a competitive labor (or land) market instead. The household inelastically supplies one unit of labor across all sectors,

$$\sum_{i=1}^{n} L_{i,t} = 1,$$

which defines the market-clearing condition for the labor market.

The household's first-order conditions are the same as in the benchmark model. On the production side, the first-order conditions of firm i are slightly different:

$$y_{ij,t} = \mu_{i,t}^{\nu} \frac{w_{ij}^{\nu} I_{i,t}}{P_{j,t}^{\nu}},$$

$$I_{i,t} = \left(\frac{\eta P_{i,t} \varepsilon_{i,t} L_{i,t}^{\psi}}{\mu_{i,t}} \right)^{\frac{1}{1-\eta}},$$

$$\mu_{i,t} = \left[\sum_{j=1}^{n} w_{ij}^{\nu} P_{j,t}^{1-\nu} \right]^{\frac{1}{1-\nu}},$$

and

$$h_t = (1-\eta) P_{i,t} \varepsilon_{i,t} I_{i,t}^{\eta} L_{i,t}^{\psi-1},$$

where h_t is the equilibrium wage (price of the additional factor). The first and third equations above are the same as before (equations (8) and (10) in the paper). The second first-order condition is slight different as the labor choice directly affects the marginal productivity of input investments. Finally, the last equation is the first-order condition on the labor investment made by sector i, which states that the marginal product of labor equals its marginal cost.

Combining the labor market clearing condition and sector i's first-order condition on labor choice, we have the equilibrium wage as a fraction of total output:

$$L_{i,t} = \frac{\psi P_{i,t} Y_{i,t}}{h_t} \text{ and } \sum_{i=1}^{n} L_{i,t} \implies h = \psi \sum_{i=1}^{n} P_{i,t} Y_{i,t}.$$

Substituting the equilibrium wage into the first-order condition on labor yields

$$L_{i,t} = \frac{P_{i,t} Y_{i,t}}{\sum_{j=1}^{n} P_{j,t} Y_{j,t}} = \delta_{i,t},$$

which means that the equilibrium labor share allocated to sector i is exactly its output share. The output shares, $\delta_{i,t}$'s, have exactly the same expression as in the benchmark model (equation (15)), because input investment expenditure is proportional to aggregate output as in the benchmark model. As a result, the same mathematical derivation from Section III.A in this Internet Appendix holds.

Following the same steps used in the derivation of Equation (16), we can solve for the output growth under the Cobb-Douglas specification, that is, $\nu = 1$. Starting from the first order condition of sector i, we have

$$(P_{i,t} Y_{i,t})^{1-\eta} = \mu_{i,t}^{-\eta} P_{i,t} \varepsilon_{i,t} \eta^{\eta} L_{i,t}^{\psi}$$

and

$$(\delta_{i,t} z_t)^{1-\eta} = \mu_{i,t}^{-\eta} P_{i,t} \varepsilon_{i,t} \eta^{\eta} L_{i,t}^{\psi},$$

which is similar to equation (16) but includes an additional labor term. If $L_{i,t} = 1$, we have equation (16) precisely.

Next, I use the fact that $L_{i,t} = \delta_{i,t}$, substitute in the multiplier expression ($\mu_{i,t}$), and take logs on both sides,

$$(\delta_{i,t} z_t)^{1-\eta} = \mu_{i,t}^{-\eta} P_{i,t} \varepsilon_{i,t} \eta^{\eta} \delta_{i,t}^{\psi},$$

$$(1-\eta-\psi)\log \delta_{i,t} + (1-\eta)\log z_t = -\eta \log \mu_{i,t} + \log P_{i,t} + \log \varepsilon_{i,t} + \eta \log \eta,$$

and

$$\begin{aligned}(1-\eta-\psi)\log \delta_{i,t} + (1-\eta)\log z_t &= -\eta \sum_j w_{ij,t} \log P_{j,t} + \eta \sum_j w_{ij,t} \log w_{ij,t} \\ &\quad + \log P_{i,t} + \log \varepsilon_{i,t} + \eta \log \eta.\end{aligned}$$

Writing the above system in matrix notation and using the price normalization yields

$$(1-\eta-\psi)\log \delta_t + (1-\eta)\mathbf{1}\log z_t = -\eta W \log P_t + \eta \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t + \log P_t + \log \varepsilon_t + \mathbf{1}\eta \log \eta,$$

which simplifies to

$$\log P_t = [\mathrm{I} - \eta W]^{-1} \left((1-\eta-\psi)\log \delta_t + (1-\eta)\mathbf{1}\log z_t - \eta \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log \varepsilon_t - \mathbf{1}\eta \log \eta \right),$$

$$\alpha' \log P_t = \alpha' [\mathrm{I} - \eta W]^{-1} \left((1-\eta-\psi)\log \delta_t + (1-\eta)\log z_t - \eta \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \log \varepsilon_t - \eta \log \eta \right),$$

$$\alpha' \log \alpha = \delta_t' \left(\frac{1-\eta-\psi}{1-\eta} \log \delta_t + \mathbf{1}\log z_t - \frac{\eta}{1-\eta} \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta} \log \varepsilon_t - \mathbf{1}\frac{\eta}{1-\eta} \log \eta \right),$$

and

$$\alpha' \log \alpha = \log z_t - \frac{\eta}{1-\eta} \log \eta + \delta_t' \left(\frac{1-\eta-\psi}{1-\eta} \log \delta_t - \frac{\eta}{1-\eta} \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta} \log \varepsilon_t \right),$$

where $\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t$ is a $n \times 1$ vector with the i^{th} element being $\mathcal{N}_{i,t}^{\mathcal{S}} = \sum_j w_{ij,t} \log w_{ij,t}$ and δ_t is a $n \times 1$ vector with output shares as in equation (15).

Thus, output in equilibrium is given by

$$\log z_t \quad = \quad \alpha' \log \alpha + \frac{\eta}{1-\eta} \log \eta - \delta_t' \left(\frac{1-\eta-\psi}{1-\eta} \log \delta_t - \frac{\eta}{1-\eta} \overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta} \log \varepsilon_t \right).$$

Taking a one period difference, equilibrium output growth is as follows

$$\log z_{t+1} - \log z_t = -\frac{1-\eta-\psi}{1-\eta} \left(\mathcal{N}_{t+1}^{\mathcal{C}} - \mathcal{N}_t^{\mathcal{C}} \right) + \frac{\eta}{1-\eta} \left(\mathcal{N}_{t+1}^{\mathcal{S}} - \mathcal{N}_t^{\mathcal{S}} \right) + \frac{1}{1-\eta} \left(e_{t+1} - e_t \right).$$

If the model with a competitive labor market features constant returns to scale, that is, $\eta + \psi = 1$, then network concentration has no effect on aggregate output growth and should not be priced. The competitive labor market model is a strong modeling assumption in which there is a production factor that is instantaneously reallocated across sectors towards those with the highest marginal product. Nevertheless, this extension emphasizes that network concentration is priced in a environment with diminishing returns to scale as some of the production factors cannot be optimally reallocated every period.

V. Output Growth: Approximation

The more general case with elasticity of substitution between inputs different from one does not have a clean closed-form solution. However, if the elasticity is different from one, the variability of inputs matters. Intuitively, the elasticity of substitution captures the ability of firms to substitute inputs. When the elasticity is greater than one, firms substitute inputs more easily and are able to substitute away less productive inputs. As a result, variability of inputs increases output in equilibrium, as firms can further specialize in using the most productive inputs. Conversely, when the elasticity is less than one, variability of inputs decreases output as firms cannot substitute away their least productive inputs. It is important to understand how the model behaves as we move away from the unit-elasticity model. To address this question, I derived the first-order approximation around the unit-elastic case, which I describe below.

Although we do not have a closed-form solution for $\nu \neq 1$, we can approximate the equilibrium solution around $\nu = 1$. The first-order approximation around $\nu = 1$ yields the following expression for consumption expenditure growth:

$$\begin{aligned}\log \mathcal{C}_{t+1} - \log \mathcal{C}_t \;\approx\;& -\left(\mathcal{N}^{\mathcal{C}}_{t+1} - \mathcal{N}^{\mathcal{C}}_t\right) + \frac{\eta}{1-\eta}\left(\mathcal{N}^{\mathcal{S}}_{t+1} - \mathcal{N}^{\mathcal{S}}_t\right) + \frac{1}{1-\eta}\left(e_{t+1} - e_t\right) \\ & + \frac{\eta}{1-\eta}\left(\delta'_{t+1}\Psi_{t+1} - \delta'_t\Psi_t\right)(\nu - 1),\end{aligned} \tag{IA.1}$$

where $\mathcal{N}^{\mathcal{C}}_t$ is the concentration factor, $\mathcal{N}^{\mathcal{S}}_t$ is the sparsity factor, e_t is the residual TFP, η measures returns to scale, $\delta_{i,t}$ is the output share of firm i when $\nu = 1$, $P_{i,t}$ is the market price of good i when $\nu = 1$, $\delta'_t\Psi_t = \sum_i \delta_{i,t}\Psi_{i,t}$, and

$$\Psi_{i,t} \equiv \frac{1}{2}\sum_j w_{ij,t}\left(\log\frac{w_{ij,t}}{P_{j,t}} - \sum_s w_{is,t}\log\frac{w_{is,t}}{P_{s,t}}\right)^2 > 0.$$

The derivation of this approximation is lengthy and is provided in detail in the next subsection.

The first three terms in equation (IA.1) are the same factors as in Theorem 1, while the last term in equation (IA.1) is the one-period change in the approximation term. Technically, the approximation term is the product of $(\nu - 1)$ and the derivative of aggregate consumption growth with respect to ν evaluated at $\nu = 1$.

The approximation term is intuitive, as the term $\Psi_{i,t}$ is proportional to the input variance when we use the network weight as the probability measure:

$$\Psi_{i,t} = \frac{1}{2}\sum_j w_{ij}\left(\log y_{ij,t} - \sum_s w_{is}\log y_{is,t}\right)^2 = \frac{1}{2}\mathrm{Var}_i\left(\log y_{ij,t}\right),$$

where $y_{ij,t}$ is the input that firm i buys from firm j in equilibrium when $\nu = 1$.[1] Therefore, the

[1] Using equation (8) from the paper and the fact that $\sum_j w_{ij} = 1$,

$$\log y_{ij,t} - \sum_s w_{is}\log y_{is,t} = \log\left(\mu_{i,t}\frac{w_{ij,t}I_{i,t}}{P_{j,t}}\right) - \sum_s w_{is}\log\left(\mu_{i,t}\frac{w_{ij,t}I_{i,t}}{P_{j,t}}\right) = \log\frac{w_{ij,t}}{P_{j,t}} - \sum_s w_{is,t}\log\frac{w_{is,t}}{P_{s,t}}.$$

term $\delta_t'\Psi_t$ is a weighted average of firm i's input dispersion. The approximation term in equation (IA.1) measures changes in the average input dispersion. If the average input dispersion increases in equilibrium, then we have

$$\delta_{t+1}'\Psi_{t+1} - \delta_t'\Psi_t > 0.$$

The effect of such a change in equilibrium consumption expenditure depends on the elasticity of substitution between inputs. If firms' production functions are more elastic and can substitute inputs more easily, that is, if $\nu > 1$, then firms invest more when input dispersion increases because they benefit from input substitution. Conversely, if firms cannot easily substitute inputs, then higher input dispersion negatively affects their total output as they cannot substitute away their least productive inputs. Therefore, input dispersion has a positive effect on the aggregate economy when firms substitute inputs more efficiently (i.e., $\nu > 1$) and a negative effect when firms cannot easily substitute inputs (i.e., $\nu < 1$).

A. Derivation

In this section, I consider an extension of the model in which the investment aggregator function is given by

$$I_{i,t} = \left[\sum_{j=1}^{n} w_{ij,t} y_{ij,t}^{1-1/\nu}\right]^{\frac{1}{1-1/\nu}}, \qquad \text{(IA.2)}$$

where ν is the elasticity of substitution between inputs.

In this framework, equilibrium output shares are given by

$$\delta_t = (1-\eta)\left[\mathbb{I} - \eta \tilde{W}_t'\right]^{-1}\alpha,$$

where $\delta_t = (\delta_{1,t}, \ldots, \delta_{n,t})'$ is a $n \times 1$ vector of output shares, $\tilde{W}_t'$ is a $n \times n$ matrix with (i, j) entry given by $\tilde{w}_{ij,t} = \frac{w_{ij,t}^{\nu} P_{j,t}^{1-\nu}}{\sum_s w_{is}^{\nu} P_s^{1-\nu}}$, and $\alpha = (\alpha_1, \ldots, \alpha_n)'$ is a $n \times 1$ vector of preference weights.

Similar to equations (13) and (16) in the paper, the following system of $n+1$ equations and $n+1$ unknowns characterize the equilibrium:

$$f_i(x,\nu) = (1-\eta)\log\delta_{i,t} + (1-\eta)\log z_t + \eta\log\mu_{i,t} - \log P_{i,t} - \log\varepsilon_{i,t} - \eta\log\eta = 0$$

for $i = 1, \ldots, n$, and

$$f_{n+1}(x,\nu) = \sum_i \alpha_i \log P_{i,t} - \sum_i \alpha_i \log\alpha_{i,t} = 0,$$

where $x = (\log P_{1,t}, \ldots, \log P_{n,t}, \log z_t)$. The function $f : \mathbb{R}^{n+2} \to \mathbb{R}^{n+1}$ specifies the system of equations that fully characterize the equilibrium. Let $x^*(\nu)$ be the solution to

$$f(x^*, \nu) = 0.$$

The equilibrium $x^*(\nu)$ depends implicitly on ν. Furthermore, we know the solution when $\nu = 1$ from the Cobb-Douglas case, and we can approximate the equilibrium $x^*(\nu)$ around $\nu = 1$

$$\log z_t \approx \log z_t\big|_{\nu=1} + \frac{\partial}{\partial\nu}\log z_t\bigg|_{\nu=1}(\nu - 1),$$

---

where the derivative term can be computed using the implicit function theorem. The term $\frac{\partial}{\partial \nu} \log z_t\big|_{\nu=1}$ is the last entry of the following $n+1 \times 1$ vector:

$$\frac{\partial}{\partial \nu} x_t^* \quad = \quad -\left[\frac{\partial}{\partial x} f(x, \nu)\right]^{-1} \frac{\partial}{\partial \nu} f(x, \nu),$$

where

$$\frac{\partial}{\partial x} f(x, \nu) = \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(x, \nu) & \cdots & \frac{\partial}{\partial x_{n+1}} f_1(x, \nu) \\ \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_1} f_{n+1}(x, \nu) & \cdots & \frac{\partial}{\partial x_{n+1}} f_{n+1}(x, \nu) \end{bmatrix}_{n+1 \times n+1}$$

$$= \begin{bmatrix} \frac{\partial}{\partial \log P_{1,t}} f_1 & \cdots & \frac{\partial}{\partial \log P_{n,t}} f_1 & \frac{\partial}{\partial \log z_t} f_1 \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial}{\partial \log P_{1,t}} f_{n+1} & \cdots & \frac{\partial}{\partial \log P_{n,t}} f_{n+1} & \frac{\partial}{\partial \log z_t} f_{n+1} \end{bmatrix}_{n+1 \times n+1},$$

and

$$\frac{\partial}{\partial \nu} f(x, \nu) = \begin{bmatrix} \frac{\partial}{\partial \nu} f_1(x, \nu) \\ \vdots \\ \frac{\partial}{\partial \nu} f_{n+1}(x, \nu) \end{bmatrix}_{n+1 \times 1}.$$

All derivatives are evaluated at $\nu = 1$.

Next, we have to calculate the entries of the matrices $\frac{\partial}{\partial x} f(x, \nu)$ and $\frac{\partial}{\partial \nu} f(x, \nu)$. Start by computing the entries of $\frac{\partial}{\partial x} f(x, \nu)$ and its inverse:

$$\frac{\partial}{\partial \log P_{j,t}} f_i\bigg|_{\nu=1} = \eta w_{ij,t} \qquad \text{for } i, j = 1, \ldots, n \text{ and } i \neq j,$$

$$\frac{\partial}{\partial \log P_{i,t}} f_i\bigg|_{\nu=1} = \eta w_{ij,t} - 1 \qquad \text{for } i = 1, \ldots, n,$$

$$\frac{\partial}{\partial \log z_t} f_i\bigg|_{\nu=1} = 1 - \eta \qquad \text{for } i = 1, \ldots, n,$$

$$\frac{\partial}{\partial \log P_{i,t}} f_{n+1}\bigg|_{\nu=1} = \alpha_i \qquad \text{for } i = 1, \ldots, n,$$

and

$$\frac{\partial}{\partial \log z_t} f_{n+1}\bigg|_{\nu=1} = 0.$$

Thus,

$$\frac{\partial}{\partial x}f(x,y) = \begin{bmatrix} \eta w_{11,t}-1 & \eta w_{12,t} & \dots & \eta w_{1n,t} & 1-\eta \\ \eta w_{21,t} & \eta w_{22,t}-1 & \dots & \eta w_{2n,t} & 1-\eta \\ \vdots & & \ddots & \vdots & \vdots \\ \eta w_{n1,t} & \eta w_{n2,t} & \dots & \eta w_{nn,t}-1 & 1-\eta \\ \alpha_1 & \alpha_2 & \dots & \alpha_n & 0 \end{bmatrix}_{n+1\times n+1}$$

$$= \begin{bmatrix} -(\mathbf{I}-\eta W) & \mathbf{1}(1-\eta) \\ \alpha' & 0 \end{bmatrix},$$

and its inverse is given by

$$\left[\frac{\partial}{\partial x}f(x,y)\right]^{-1} = \begin{bmatrix} -(\mathbf{I}-\eta W) & \mathbf{1}(1-\eta) \\ \alpha & 0 \end{bmatrix}^{-1} = \begin{bmatrix} -(\mathrm{I}-\mathbf{1}\alpha')(\mathbf{I}-\eta W)^{-1} & \mathbf{1} \\ \alpha'(\mathbf{I}-\eta W)^{-1} & 1 \end{bmatrix}.$$

Next, compute the entries of $\frac{\partial}{\partial \nu}f(x,\nu)$:

$$\frac{\partial}{\partial \nu}f_i\bigg|_{\nu=1} = (1-\eta)\,\frac{\partial}{\partial \nu}\log\delta_{i,t}\bigg|_{\nu=1} + \eta\,\frac{\partial}{\partial \nu}\log\mu_{i,t}\bigg|_{\nu=1} \qquad \text{for } i=1,\dots,n,$$

and

$$\frac{\partial}{\partial \nu}f_{n+1}\bigg|_{\nu=1} = 0,$$

where

$$\begin{aligned}
\frac{\partial}{\partial \nu}\log\mu_{i,t}\bigg|_{\nu=1} &= \frac{\partial}{\partial \nu}\frac{\log\left(\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}\right)}{1-\nu}\Bigg|_{\nu=1} \\
&= \lim_{\nu\to1}\frac{\partial}{\partial \nu}\frac{\log\left(\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}\right)}{1-\nu} \\
&= \lim_{\nu\to1}\frac{1}{(1-\nu)^2}\left[\frac{(1-\nu)\left(\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}\left(\log w_{ij,t}-\log P_{j,t}\right)\right)}{\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}}\right. \\
&\qquad\left. +\log\left(\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}\right)\right] \\
&= \lim_{\nu\to1}\frac{\sum_j(1-\nu)\tilde{w}_{ij,t}\log\frac{w_{ij,t}}{P_{j,t}} + \log\left(\sum_j w_{ij,t}^{\nu}P_{j,t}^{1-\nu}\right)}{(1-\nu)^2},
\end{aligned}$$

and $\tilde{w}_{ij,t} = \frac{w_{ij,t}^{\nu} P_{j,t}^{1-\nu}}{\sum_j w_{ij,t}^{\nu} P_{j,t}^{1-\nu}}$. Using L'Hôspital's rule twice, we have

$$
\begin{aligned}
\frac{\partial}{\partial \nu} \log \mu_{i,t}\Big|_{\nu=1} &= \lim_{\nu \to 1} \frac{\sum_j (1-\nu)\tilde{w}_{ij,t} \log \frac{w_{ij,t}}{P_{j,t}} + \log\left(\sum_j w_{ij,t}^{\nu} P_{j,t}^{1-\nu}\right)}{(1-\nu)^2} \\
&= \lim_{\nu \to 1} \frac{\sum_j \left((1-\nu)\frac{\partial \tilde{w}_{ij,t}}{\partial \nu} - \tilde{w}_{ij,t}\right) \log \frac{w_{ij,t}}{P_{j,t}} + \frac{\sum_j w_{ij,t}^{\nu} P_{j,t}^{1-\nu} \log \frac{w_{ij,t}}{P_{j,t}}}{\sum_j w_{ij,t}^{\nu} P_{j,t}^{1-\nu}}}{-2(1-\nu)} \\
&= \lim_{\nu \to 1} \frac{\sum_j \left((1-\nu)\frac{\partial \tilde{w}_{ij,t}}{\partial \nu} - \tilde{w}_{ij,t}\right) \log \frac{w_{ij,t}}{P_{j,t}} + \sum_j \tilde{w}_{ij,t} \log \frac{w_{ij,t}}{P_{j,t}}}{-2(1-\nu)} \\
&= \lim_{\nu \to 1} \frac{\sum_j \left((1-\nu)\frac{\partial^2 \tilde{w}_{ij,t}}{\partial \nu^2} - 2\frac{\partial \tilde{w}_{ij,t}}{\partial \nu}\right) \log \frac{w_{ij,t}}{P_{j,t}} + \sum_j \frac{\partial \tilde{w}_{ij,t}}{\partial \nu} \log \frac{w_{ij,t}}{P_{j,t}}}{2} \\
&= \lim_{\nu \to 1} \frac{\sum_j \left((1-\nu)\frac{\partial^2 \tilde{w}_{ij,t}}{\partial \nu^2} - \frac{\partial \tilde{w}_{ij,t}}{\partial \nu}\right) \log \frac{w_{ij,t}}{P_{j,t}}}{2} \\
&= \frac{1}{2} \lim_{\nu \to 1} \sum_j \left((1-\nu)\frac{\partial^2 \tilde{w}_{ij,t}}{\partial \nu^2} - \frac{\partial \tilde{w}_{ij,t}}{\partial \nu}\right) \log \frac{w_{ij,t}}{P_{j,t}} \\
&= -\frac{1}{2} \lim_{\nu \to 1} \sum_j \left(\frac{\partial \tilde{w}_{ij,t}}{\partial \nu}\right) \log \frac{w_{ij,t}}{P_{j,t}} \\
&= -\frac{1}{2} \sum_j w_{ij,t} \left(\log \frac{w_{ij,t}}{P_{j,t}} - \sum_s w_{is,t} \log \frac{w_{is,t}}{P_{s,t}}\right) \log \frac{w_{ij,t}}{P_{j,t}} \\
&= -\frac{1}{2} \sum_j w_{ij,t} \left(\log \frac{w_{ij,t}}{P_{j,t}} - \sum_s w_{is,t} \log \frac{w_{is,t}}{P_{s,t}}\right)^2 = -\Psi_{i,t},
\end{aligned}
$$

where $P_{j,t}$ is evaluated at $\nu = 1$.

The term $\frac{\partial}{\partial \nu} \log \delta_{i,t}\big|_{\nu=1}$ has to be computed using the implicit function theorem as well. The output shares, $\{\delta_{i,t}\}_i$, are the solution to the following system of equations:

$$g_i(\delta_t, \nu) = 0 \;\; \forall \;\; i = 1, \ldots, n,$$

where

$$g_i(\delta_t, \nu) = (1-\eta)\alpha_i + \eta \sum_j \tilde{w}_{ji,t} \delta_{j,t} - \delta_{i,t}.$$

Therefore,

$$
\begin{aligned}
\frac{\partial}{\partial \nu} \delta_t &= -\left[\frac{\partial}{\partial \delta_t} g(\delta_t, \nu)\right]^{-1} \frac{\partial}{\partial \nu} g(\delta_t, \nu) \\
&= \left[\mathrm{I} - \eta \tilde{W}'\right]^{-1} \begin{bmatrix} \eta \sum_j \delta_{j,t} \frac{\partial}{\partial \nu} \tilde{w}_{j1,t} \\ \vdots \\ \eta \sum_j \delta_{j,t} \frac{\partial}{\partial \nu} \tilde{w}_{jn,t} \end{bmatrix},
\end{aligned}
$$

where

$$\frac{\partial}{\partial\nu}\tilde{w}_{ij,t}\bigg|_{\nu=1} \quad = \quad w_{ij,t}\left(\log\frac{w_{ij,t}}{P_{j,t}} - \sum_s w_{is}\log\frac{w_{is,t}}{P_{s,t}}\right) \equiv \bar{w}_{ij,t}.$$

Notice that $\sum_j \bar{w}_{ij} = 0$ and $\sum_j \frac{\partial}{\partial\nu}\delta_{j,t}\big|_{\nu=1} = 0$. Let $\bar{W}_t$ be a $n\times n$ matrix whose (i,j) entry is $\bar{w}_{ij,t}$. Then

$$\frac{\partial}{\partial\nu}\delta_t\bigg|_{\nu=1} = \left[\mathrm{I}-\eta W_t'\right]^{-1}\bar{W}_t'\delta_t,$$

$$\frac{\partial}{\partial\nu}\log\delta_{i,t}\bigg|_{\nu=1} = \frac{1}{\delta_{i,t}}\frac{\partial}{\partial\nu}\delta_{i,t}\bigg|_{\nu=1},$$

and

$$\frac{\partial}{\partial\nu}f(x,\nu) = \begin{bmatrix} (1-\eta)\frac{1}{\delta_{1,t}}\frac{\partial}{\partial\nu}\delta_{1,t}\big|_{\nu=1} - \eta\Psi_{1,t} \\ \vdots \\ (1-\eta)\frac{1}{\delta_{n,t}}\frac{\partial}{\partial\nu}\delta_{n,t}\big|_{\nu=1} - \eta\Psi_{n,t} \\ 0 \end{bmatrix},$$

which yields

$$\begin{aligned}\frac{\partial}{\partial\nu}x_t^* &= -\left[\frac{\partial}{\partial x}f(x,\nu)\right]^{-1}\frac{\partial}{\partial\nu}f(x,\nu) \\ &= -\begin{bmatrix} -(\mathrm{I}-\mathbf{1}\alpha')(\mathbf{I}-\eta W)^{-1} & \mathbf{1} \\ \alpha'(\mathbf{I}-\eta W)^{-1} & 1\end{bmatrix}\begin{bmatrix} (1-\eta)\frac{1}{\delta_{1,t}}\frac{\partial}{\partial\nu}\delta_{1,t}\big|_{\nu=1} - \eta\Psi_{1,t} \\ \vdots \\ (1-\eta)\frac{1}{\delta_{n,t}}\frac{\partial}{\partial\nu}\delta_{n,t}\big|_{\nu=1} - \eta\Psi_{n,t} \\ 0 \end{bmatrix},\end{aligned}$$

and, since $\frac{\partial}{\partial \nu} \log z_t$ is the last entry of the vector above,

$$
\begin{aligned}
\frac{\partial}{\partial \nu} \log z_t &= -\left[\begin{array}{cc} \alpha'(\mathbf{I}-\eta W)^{-1} & 1 \end{array}\right] \left[\begin{array}{c} (1-\eta)\frac{1}{\delta_{1,t}} \frac{\partial}{\partial \nu}\delta_{1,t}\big|_{\nu=1} - \eta\Psi_{1,t} \\ \vdots \\ (1-\eta)\frac{1}{\delta_{n,t}} \frac{\partial}{\partial \nu}\delta_{n,t}\big|_{\nu=1} - \eta\Psi_{n,t} \\ 0 \end{array}\right] \\
&= -\left[\begin{array}{cc} \frac{1}{1-\eta}\delta_t' & 1 \end{array}\right] \left[\begin{array}{c} (1-\eta)\frac{1}{\delta_{1,t}} \frac{\partial}{\partial \nu}\delta_{1,t}\big|_{\nu=1} - \eta\Psi_{1,t} \\ \vdots \\ (1-\eta)\frac{1}{\delta_{n,t}} \frac{\partial}{\partial \nu}\delta_{n,t}\big|_{\nu=1} - \eta\Psi_{n,t} \\ 0 \end{array}\right] \\
&= -\sum_j \frac{1}{1-\eta}\delta_{j,t}\left((1-\eta)\frac{1}{\delta_{j,t}}\frac{\partial}{\partial \nu}\delta_{j,t}\bigg|_{\nu=1} - \eta\Psi_{j,t}\right) \\
&= -\sum_j \frac{1}{1-\eta}\delta_{j,t}\left((1-\eta)\frac{1}{\delta_{j,t}}\frac{\partial}{\partial \nu}\delta_{j,t}\bigg|_{\nu=1} - \eta\Psi_{j,t}\right) \\
&= -\sum_j \frac{\partial}{\partial \nu}\delta_{j,t}\bigg|_{\nu=1} + \frac{\eta}{1-\eta}\sum_j \delta_{j,t}\Psi_{j,t} \\
&= \frac{\eta}{1-\eta}\sum_j \delta_{j,t}\Psi_{j,t} \\
&= \frac{\eta}{1-\eta}\delta_t'\Psi_t,
\end{aligned}
$$

where δ_t is evaluated at $\nu = 1$.

Hence, output can be approximated by

$$
\begin{aligned}
\log z_t &\approx \log z_t|_{\nu=1} + \frac{\partial}{\partial \nu}\log z_t\bigg|_{\nu=1}(\nu - 1) \\
&= \log z_t|_{\nu=1} + \frac{\eta}{1-\eta}\delta_t'\Psi_t(\nu-1) \\
&= \alpha'\log\alpha + \frac{\eta}{1-\eta}\log\eta - \delta_t'\left(\log\delta_t - \frac{\eta}{1-\eta}\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t - \frac{1}{1-\eta}\log\varepsilon_t\right) + \frac{\eta}{1-\eta}\delta_t'\Psi_t(\nu-1),
\end{aligned}
$$

and the approximation for output growth is given by

$$
\begin{aligned}
\log z_{t+1} - \log z_t &\approx -\left(\delta_{t+1}'\log\delta_{t+1} - \delta_t'\log\delta_t\right) + \frac{\eta}{1-\eta}\left(\delta_{t+1}'\overrightarrow{\mathcal{N}^{\mathcal{S}}}_{t+1} - \delta_t'\overrightarrow{\mathcal{N}^{\mathcal{S}}}_t\right) \\
&\quad + \frac{1}{1-\eta}\left(\delta_{t+1}'\log\varepsilon_{t+1} - \delta_t'\log\varepsilon_t\right) + \frac{\eta}{1-\eta}\left(\delta_{t+1}'\Psi_{t+1} - \delta_t'\Psi_t\right)(\nu-1) \\
&= -\left(\mathcal{N}_{t+1}^{\mathcal{C}} - \mathcal{N}_t^{\mathcal{C}}\right) + \frac{\eta}{1-\eta}\left(\mathcal{N}_{t+1}^{\mathcal{S}} - \mathcal{N}_t^{\mathcal{S}}\right) + \frac{1}{1-\eta}\left(e_{t+1} - e_t\right) \\
&\quad + \frac{\eta}{1-\eta}\left(\delta_{t+1}'\Psi_{t+1} - \delta_t'\Psi_t\right)(\nu-1).
\end{aligned}
$$

VI. Calibration

In this section, I verify whether the multisector network model is quantitatively consistent with the empirical evidence on return spreads. The model is calibrated to replicate the sorted portfolios from Table II, as well as other asset pricing moments. First, I add more structure to the model to solve for the SDF. I then discuss the calibration procedure.[2]

A. Setup

The general equilibrium model specifies how innovations in network factors affect aggregate consumption (equation (17)). However, a consumption claim in the model does not have leverage, and, to calibrate the model, both the levered and unlevered consumption processes have to be specified. First, I specify the unlevered consumption claim by

$$\begin{aligned}\log z_{t+1} - \log z_t &= \phi_{\mathcal{N}^\mathcal{C}} \Delta \mathcal{N}^\mathcal{C}_{t+1} + \phi_{\mathcal{N}^\mathcal{S}} \Delta \mathcal{N}^\mathcal{S}_{t+1} + \phi_e \Delta e_{t+1} + \phi_x x_t, \\ \Delta \mathcal{N}^\mathcal{C}_{t+1} &= \sigma_{\mathcal{N}^\mathcal{C}} \varepsilon_{\mathcal{N}^\mathcal{C},t+1}, \\ \Delta \mathcal{N}^\mathcal{S}_{t+1} &= \sigma_{\mathcal{N}^\mathcal{S}} \varepsilon_{\mathcal{N}^\mathcal{S},t+1}, \\ \Delta e_{t+1} &= \sigma_e \varepsilon_{e,t+1},\end{aligned}$$

and

$$x_{t+1} = \rho_x x_t + \sigma_x \varepsilon_{x,t+1},$$

where $\varepsilon_{e,t+1}$, $\varepsilon_{\mathcal{N}^\mathcal{C},t+1}$, $\varepsilon_{\mathcal{N}^\mathcal{S},t+1}$, and $\varepsilon_{x,t+1}$ are i.i.d. standard normal random variables. In this specification I assume that innovations in both network factors are i.i.d. shocks, which is consistent with the data. The expression for unlevered consumption has an extra term, x_t, which is a long-run risk factor based on Bansal and Yaron (2004). The long-run risk term is interpreted as a persistent component of expected consumption growth and is included in the unlevered consumption claim to generate an equity risk premium consistent with the data. In the calibrated model, the long-run risk factor is quantitatively irrelevant to generate return spreads in the cross-section of sorted portfolios. It is included in the model only to generate reasonable asset pricing moments for the market portfolio. In the model, $\log z_{t+1} - \log z_t$ is the growth rate of the unlevered consumption claim and loadings are given by

$$\phi_{\mathcal{N}^\mathcal{C}} = -\frac{1}{1-\eta}, \quad \phi_{\mathcal{N}^\mathcal{S}} = \frac{\eta}{1-\eta}, \quad \text{and} \quad \phi_e = \frac{1}{1-\eta},$$

where η is the returns to scale.

The first-order approximation of the return on total wealth is given by

$$r^W_{t+1} = \kappa^c_0 + \Delta \log z_{t+1} + wc_{t+1} - \kappa^c_1 wc_t,$$

where $\kappa^c_0 = -\log\left(\exp(\mu_{wc}) - 1\right) + \frac{\exp(\mu_{wc})}{\exp(\mu_{wc})-1}\mu_{wc}$, $\kappa^c_1 = \frac{\exp(\mu_{wc})}{\exp(\mu_{wc})-1} > 1$, and μ_{wc} is the unconditional average of the wealth consumption expenditure ratio. One can guess that the wealth-consumption expenditure ratio is linear on the long-run risk term:

$$wc_t = \mu_{wc} + A x_t,$$

where μ_{wc} and $A = \frac{\phi_x(1-\rho)}{\kappa^c_1 - \rho_x}$ are constants determined using the representative household's Euler

[2] All derivations and expressions are detailed in Sections VI.D and VI.E in this Internet Appendix.

equation. The approximation of the return on total wealth then becomes

$$r_{t+1}^{W} = r_0^c + \beta_x x_t + \beta_{\varepsilon e}\sigma_e\varepsilon_{e,t+1} + \beta_{\varepsilon x}\sigma_x\varepsilon_{x,t+1} + \beta_{\varepsilon\mathcal{N}^C}\sigma_{\mathcal{N}^C}\varepsilon_{\mathcal{N}^C,t+1} + \beta_{\varepsilon\mathcal{N}^S}\sigma_{\mathcal{N}^S}\varepsilon_{\mathcal{N}^S,t+1},$$

where $r_0^c = \kappa_0^c - \mu_{wc}(\kappa_1^c - 1)$, $\beta_x = \rho\phi_x$, $\beta_{\varepsilon e} = \phi_e$, $\beta_{\varepsilon x} = A$, $\beta_{\varepsilon\mathcal{N}^C} = \phi_{\mathcal{N}^C}$, and $\beta_{\varepsilon\mathcal{N}^S} = \phi_{\mathcal{N}^S}$.

The SDF is derived based on unlevered consumption claim and is given by

$$m_{t+1} = \mu_s + \lambda_x x_t - \lambda_{\varepsilon e}\sigma_e\varepsilon_{e,t+1} - \lambda_{\varepsilon x}\sigma_x\varepsilon_{x,t+1} - \lambda_{\varepsilon\mathcal{N}^C}\sigma_{\mathcal{N}^C}\varepsilon_{\mathcal{N}^C,t+1} - \lambda_{\varepsilon\mathcal{N}^S}\sigma_{\mathcal{N}^S}\varepsilon_{\mathcal{N}^S,t+1},$$

where

$$\begin{aligned}
\mu_s &= \theta\log\beta + (\theta-1)(\kappa_0^c - \mu_{wc}(\kappa_1^c - 1)),\\
\lambda_x &= -\gamma\phi_e\phi_x - (\theta-1)A(\kappa_1^c - \rho_x) = -\rho\phi_x,\\
\lambda_{\varepsilon e} &= \gamma\phi_e,\\
\lambda_{\varepsilon x} &= -(\theta-1)A = (\gamma-\rho)\frac{\phi_x}{\kappa_1^c - \rho_x},\\
\lambda_{\varepsilon\mathcal{N}^C} &= \gamma\phi_{\mathcal{N}^C},
\end{aligned}$$

and

$$\lambda_{\varepsilon\mathcal{N}^S} = \gamma\phi_{\mathcal{N}^S}.$$

The prices of risk of network concentration and sparsity are $\lambda_{\varepsilon\mathcal{N}^C}$ and $\lambda_{\varepsilon\mathcal{N}^S}$, respectively. Thus, innovations in network sparsity carry a positive price of risk, while innovations in network concentration carry a negative price of risk, since $\phi_{\mathcal{N}^C} < 0$ and $\phi_{\mathcal{N}^S} > 0$.

The levered consumption claim, that is, market dividend, may have loadings different from the unlevered claim. In a more general expression, let the growth rate of the levered consumption claim be given by:

$$\Delta d_t^m = \phi_x^m x_t + \varphi_{\mathcal{N}^C}^m\sigma_{\mathcal{N}^C}\varepsilon_{\mathcal{N}^C,t+1} + \varphi_{\mathcal{N}^S}^m\sigma_{\mathcal{N}^S}\varepsilon_{\mathcal{N}^S,t+1} + \varphi_e^m\sigma_e\varepsilon_{e,t+1} + \varphi_\xi^m\sigma_\xi\varepsilon_{\xi,t+1},$$

where the coefficients ϕ_x^m, φ_V^m, φ_E^m, φ_e^m, and φ_ξ^m are calibrated to match market return properties, such as betas and the equity risk premium. The process of leveraging the consumption claim affects the exposure to the asset pricing factors. Thus, levered consumption may have betas different from the unlevered claim.

For a portfolio dividend, let portfolio i's dividend growth process be described by

$$\Delta d_{i,t+1} = \mu^i + \phi_x^i x_t + \varphi_e^i\sigma_e\varepsilon_{e,t+1} + \varphi_x^i\sigma_x\varepsilon_{x,t+1} + \varphi_{\mathcal{N}^C}^i\sigma_{\mathcal{N}^C}\varepsilon_{\mathcal{N}^C,t+1} + \varphi_{\mathcal{N}^S}^i\sigma_{\mathcal{N}^S}\varepsilon_{\mathcal{N}^S,t+1},$$

which is the projection of stock i's dividend onto the asset pricing factors. Similar to the market portfolio, the coefficients of the dividend growth process are calibrated to match the asset pricing moments of the portfolios.

Finally, to compute the return of the portfolio i, I guess and verify that its log price-dividend ratio is linear in the long-run risk term. Then,

$$pd_{i,t} = \mu_{pd}^i + A_i x_t,$$

where μ_{pd}^i and $A_i = \frac{\phi_x^i + \lambda_x}{1-\kappa_1^i\rho_x}$ are constants determined by the Euler equation. The return of portfolio i is approximated by

$$r_{i,t+1} = \Delta d_{i,t+1} + \kappa_0^i + \kappa_1^i pd_{i,t+1} - pd_{i,t},$$

Table IA.I
Parameters

This table reports the parameters used in the calibrated model. Panel A shows residual TFP innovations' standard deviation (σ_e), concentration innovations' standard deviation ($\sigma_{\mathcal{N}^C}$), and sparsity innovations' standard deviation ($\sigma_{\mathcal{N}^C}$). Panel B reports the preference parameters over risk aversion, elasticity of intertemporal substitution ($1/pho$), and the intertemporal discount rate (β). Finally, Panel C reports returns to scale (η). The calibration is at a monthly frequency.

Parameters	Description	Value
	Panel A: Factors' volatility (annualized values)	
σ_e	Residual TFP innovations' standard deviation	0.0114
$\sigma_{\mathcal{N}^C}$	Concentration innovations' standard deviation	0.0138
$\sigma_{\mathcal{N}^S}$	Sparsity innovations standard deviation	0.0277
	Panel B: Preferences	
γ	Risk aversion	10
$1/\rho$	Elasticity of intertemporal substitution	1.50
β	Discount rate (monthly)	0.996
	Panel C: Technology	
η	Returns to scale	0.35

where $\kappa_1^i = \frac{\exp(\mu_{pd}^i)}{1+\exp(\mu_{pd}^i)}$ and $\kappa_0^i = \log\left(1+\exp\left(\mu_{pd}^i\right)\right) - \kappa_1^i\mu_{pd}^i$ are the approximation constants. Substituting the price-dividend expression into the return approximation yields

$$r_{i,t+1} = r_0^i + \beta_{i,x}x_t + \beta_{i,e\varepsilon}\sigma_e\varepsilon_{e,t+1} + \beta_{i,x\varepsilon}\sigma_x\varepsilon_{x,t+1} + \beta_{i,\mathcal{N}^C\varepsilon}\sigma_{\mathcal{N}^C}\varepsilon_{\mathcal{N}^C,t+1} + \beta_{i,\mathcal{N}^S\varepsilon}\sigma_{\mathcal{N}^S}\varepsilon_{\mathcal{N}^S,t+1},$$

where $r_0^i = \mu^i + \mu_{pd}^i(\kappa_1^i - 1) + \kappa_0^i$, $\beta_x^i = \rho\phi_e\phi_x$, $\beta_{\varepsilon e}^i = \varphi_e^i$, $\beta_{\varepsilon x}^i = \varphi_x^i + \kappa_1^i A_i$, $\beta_{\varepsilon\mathcal{N}^C}^i = \varphi_{\mathcal{N}^C}^i$, and $\beta_{\varepsilon\mathcal{N}^S}^i = \varphi_{\mathcal{N}^S}^i$. Similar procedure follows for the market return.

B. Parameters

I calibrate the model at the monthly frequency. The calibrated parameters are in Table IA.I. Next, I describe the calibration of the unlevered consumption and preferences, the market portfolio, and the sorted portfolios.

The long-run risk term and preference calibration is based on Bansal and Yaron (2004). The long-run term x_t is highly persistent ($\rho_x = 0.979$) and its innovations are not too volatile ($\sigma_x = 0.0044 \times 0.0078 \approx 3.43 \times 10^{-4}$). Risk aversion is $\gamma = 10$ and intertemporal elasticity of substitution is above one ($\frac{1}{\rho} = 1.5$). The parameter ϕ_x is calibrated at 1.65 for the model to generate an equity premium similar to the data. The discount rate β is calibrated to match the average risk-free rate of return of 1.38% per year observed in the data.

The volatility of the network factors ($\sigma_{\mathcal{N}^C}$ and $\sigma_{\mathcal{N}^S}$) are calibrated to match the standard deviations of the network factors. I can match the volatility of sparsity innovation exactly. The volatility of the innovation in network concentration based on the Compustat data is 0.0277, which would generate a very volatile consumption process. Using BEA data, the volatility of the innovation in network concentration is considerably lower (0.0055). To reconcile these two sources of data and keep the consumption volatility compatible with the data, I set the standard deviation of innovation

Table IA.II
Model-Implied Asset Pricing Moments

This table reports several model-implied asset pricing moments. Panel A reports the average volatility of the risk-free rate of return. Panel B reports the equity risk premium (ERP) in the model and in the data, as well as the model ERP decomposition between residual TFP (ε_e), long-run risk (ε_x), concentration ($\varepsilon_{\mathcal{N}^C}$), and sparsity ($\varepsilon_{\mathcal{N}^S}$). Panel C reports return volatility in the model and in the data. Panel D reports model-implied betas, and Panel E reports the calibrated dividend parameters. Columns (1) and (2) report moments for the unlevered and levered (market) consumption claims. Columns (3) to (8) report asset moments for the sparsity- and concentration-beta-sorted portfolios, from low (L) to high (H) network beta.

	Unlevered	Market	Sparsity Sorted			Concentration Sorted		
	Consumption		L		H	L		H
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk-free return								
Mean rf (%)	–	1.38	–	–	–	–	–	–
rf vol (%)	–	0.64	–	–	–	–	–	–
Panel B: ERP decomposition								
Data (%)	–	7.73	6.42	8.35	11.03	10.28	7.82	7.10
Model (%)	2.50	7.76	4.57	7.03	9.75	9.79	7.31	5.08
ε_e	0.31	0.62	0.52	0.25	1.48	0.96	0.24	0.73
ε_x	1.78	6.89	6.89	6.89	6.89	6.89	6.89	6.89
$\varepsilon_{\mathcal{N}^C}$	0.19	0.08	−0.30	0.30	−0.31	2.12	0.65	−1.48
$\varepsilon_{\mathcal{N}^S}$	0.22	0.17	−2.54	−0.42	1.70	−0.18	−0.47	−1.06
Panel C: Volatility								
Data (%)	–	16.89	16.48	13.41	14.46	15.65	13.21	15.25
Model (%)	3.68	16.03	19.95	10.47	17.36	19.04	11.34	16.68
Panel D: Betas								
β_e	1.54	3.10	2.63	1.27	7.40	4.81	1.19	3.66
β_x	21.24	81.95	81.95	81.95	81.95	81.95	81.95	81.95
$\beta_{\mathcal{N}^C}$	−1.00	−0.44	1.59	−1.59	1.65	−11.14	−3.40	7.79
$\beta_{\mathcal{N}^S}$	0.54	0.40	−6.14	−1.01	4.12	−0.43	−1.13	−2.56
Panel E: Dividend parameters								
$\mu^i(\times 100)$	–	0.00	0.05	0.03	0.08	0.14	0.03	0.03
ϕ_x^i	–	3.50	3.50	3.50	3.50	3.50	3.50	3.50
φ_x^i	–	0.00	−12.08	−0.84	1.80	−0.55	−0.33	−7.98
φ_e^i	–	3.10	2.63	1.27	7.40	4.81	1.19	3.66
$\varphi_{\mathcal{N}^C}^i$	–	−0.44	1.59	−1.59	1.65	−11.14	−3.40	7.79
$\varphi_{\mathcal{N}^S}^i$	–	0.40	−6.14	−1.01	4.12	−0.43	−1.13	−2.56
φ_ξ^i	–	4.50	0.00	0.00	0.00	0.00	0.00	0.00

in the network concentration at an intermediate value of $\sigma_{\mathcal{N}^C} = 0.0138$ (annualized). Finally, σ_e is calibrated to match the consumption growth rate volatility at 2.85% (annual), and the returns to scale to capital is set at $\eta = 0.35$ following the literature.

Following Bansal and Yaron (2004), the leverage parameter is calibrated at $\phi_x^m = 3.5$, and the parameters of the idiosyncratic component of the market dividend are calibrated at $\varphi_\xi^m = 4.5$ and

$\sigma_\xi = 0.0078$ to generate reasonable dividend growth volatility, average excess return, and return volatility. The loading on the factors' innovations (φ^m_V, φ^m_E, and φ^m_e) are calibrated to match the factor betas of the market portfolio.

For the portfolio dividends growth process, the loadings on the long-run term are the same as the market portfolio ($\phi^i_x = \phi^m_x = 3.5$) and the loading on its innovation (φ^i_x) matches the $\beta_{\varepsilon x}$ of the market portfolio precisely. This calibration strategy makes the contribution of the long-run risk to the risk premium constant across all calibrated portfolios. Thus, all of the equity risk premium spreads are due to other risk factors, not the long-run risk. This guarantees that the long-run risk is quantitatively irrelevant to generate the return spreads observed in the data. Most importantly, φ^i_V, φ^i_E, and φ^i_e are calibrated to match the betas estimated from the data.

C. *Calibrated model*

Table IA.II reports asset pricing moments for the market portfolio as well as for the calibrated portfolios. Panel A shows the average and the volatility of the risk-free rate of return. The average is the same as in the data, but the model generates a risk-free rate of return not as volatile as in the data: in the model its volatility is 0.64%, while in the data it is between 1% to 2% depending on the sample considered. Panels B and C report the risk premium and return volatility, respectively. Finally, Panel D reports portfolio factor betas and Panel E reports the parameterization for each portfolio.

The model replicates the expected return spread in the calibrated portfolios. The model generates a spread of 5.18% in the sparsity-beta-sorted portfolios, while the spread in the data is 4.61%. The model-implied return spread in the concentration-beta-sorted portfolio is -4.71%, which is comparable to the -3.18% spread we observe in the data. The return volatilities are also replicated, and the model-implied volatility is close to what we observe in the data.

The contribution of the long-run risk factor to the risk premium is constant across all portfolios, which means that long-run risk does not affect the return spread. All of the return spreads come from having different exposures to the factors. The 5.18% return spread in the sparsity-beta-sorted portfolios, 82% (4.24 percentage points out of the 5.18% spread) is due to different exposures (betas) to innovations in the network sparsity factor. For the concentration-beta-sorted portfolios, 76% (3.60 percentage points out of the 4.71% spread) is due to different exposures (betas) to innovations in the network concentration factor. Thus, the driving force behind the return spread is the different exposures to innovations in the network factors.

D. *Stochastic Discount Factor Derivation*

In this section, I provide a detailed derivation of the wealth consumption expenditure ratio, the SDF and the prices of risk. The model can be fully described by

$$\begin{aligned}
\log \mathcal{C}_{t+1} - \log \mathcal{C}_t &= \phi_{\mathcal{N}^\mathcal{C}} \left(\mathcal{N}^\mathcal{C}_{t+1} - \mathcal{N}^\mathcal{C}_t \right) + \phi_{\mathcal{N}^\mathcal{S}} \left(\mathcal{N}^\mathcal{S}_{t+1} - \mathcal{N}^\mathcal{S}_t \right) + \phi_x x_t + \phi_e \sigma_e \varepsilon_{e,t+1} \qquad \text{(IA.3)} \\
\mathcal{N}^\mathcal{C}_{t+1} - \mathcal{N}^\mathcal{C}_t &= \sigma_{\mathcal{N}^\mathcal{C}} \varepsilon_{\mathcal{N}^\mathcal{C},t+1} \\
\mathcal{N}^\mathcal{S}_{t+1} - \mathcal{N}^\mathcal{S}_t &= \sigma_{\mathcal{N}^\mathcal{S}} \varepsilon_{\mathcal{N}^\mathcal{S},t+1} \\
x_{t+1} &= \rho_x x_t + \sigma_x \varepsilon_{x,t+1},
\end{aligned}$$

where $\varepsilon_{e,t+1}$, $\varepsilon_{x,t+1}$, $\varepsilon_{\mathcal{N}^\mathcal{C},t+1}$, and $\varepsilon_{\mathcal{N}^\mathcal{S},t+1}$ are i.i.d. standard normal random variables.

The return on total wealth can be approximated (first order approximation) by

$$r^W_{t+1} = \kappa^c_0 + \Delta\omega_{t+1} + wc_{t+1} - \kappa^c_1 wc_t, \qquad \text{(IA.4)}$$

where $\Delta\omega_{t+1} = \log \mathcal{C}_{t+1} - \log \mathcal{C}_t$,

$$\begin{aligned}\kappa_0^c &= -\log\left(\exp(\mu_{wc}) - 1\right) + \frac{\exp(\mu_{wc})}{\exp(\mu_{wc}) - 1}\mu_{wc}, \\ \kappa_1^c &= \frac{\exp(\mu_{wc})}{\exp(\mu_{wc}) - 1} > 1,\end{aligned}$$

and μ_{wc} is the unconditional average of the wealth consumption expenditure ratio.

Guess that the wealth-consumption expenditure ratio is linear in the long-run risk factor:

$$wc_t = \mu_{wc} + Ax_t, \tag{IA.5}$$

where A is a constant.

We can derive the return on total wealth by substituting equations (IA.3) and (IA.5) into the expression for the return on total wealth given by equation (IA.4):

$$\begin{aligned}r_{t+1}^W &= \kappa_0^c + \Delta\omega_{t+1} + wc_{t+1} - \kappa_1^c wc_t \\ &= r_0^c + \beta_x x_t + \beta_{\varepsilon e}\sigma_e \varepsilon_{e,t+1} + \beta_{\varepsilon x}\sigma_x \varepsilon_{x,t+1} \\ &\qquad + \beta_{\varepsilon \mathcal{N}^c}\sigma_{\mathcal{N}^c}\varepsilon_{\mathcal{N}^c,t+1} + \beta_{\varepsilon \mathcal{N}^s}\sigma_{\mathcal{N}^s}\varepsilon_{\mathcal{N}^s,t+1},\end{aligned}$$

where

$$r_0^c = \kappa_0^c + \mu_{wc}(1 - \kappa_1^c) \tag{IA.6}$$
$$\beta_x = \phi_x - A(\kappa_1^c - \rho_x) \tag{IA.7}$$
$$\beta_{\varepsilon x} = A \tag{IA.8}$$
$$\beta_{\varepsilon e} = \phi_e \tag{IA.9}$$
$$\beta_{\varepsilon \mathcal{N}^c} = \phi_{\mathcal{N}^c} \tag{IA.10}$$
$$\beta_{\varepsilon \mathcal{N}^s} = \phi_{\mathcal{N}^s}. \tag{IA.11}$$

Next, we can write the SDF as follows:

$$\begin{aligned}m_{t+1} &= \theta \log \beta - \rho\theta\Delta\omega_{t+1} + (\theta - 1)r_{t+1}^{\mathcal{W}} \\ &= \mu_s + \lambda_x x_t - \lambda_{\varepsilon e}\sigma_e \varepsilon_{e,t+1} - \lambda_{\varepsilon x}\sigma_x \varepsilon_{e,t+1} \\ &\qquad - \lambda_{\varepsilon \mathcal{N}^c}\sigma_{\mathcal{N}^c}\varepsilon_{\mathcal{N}^c,t+1} - \lambda_{\varepsilon \mathcal{N}^s}\sigma_{\mathcal{N}^s}\varepsilon_{\mathcal{N}^s,t+1},\end{aligned}$$

where $\theta = \frac{1-\gamma}{1-\rho}$,

$$\mu_s = \theta \log \beta + (\theta - 1)(\kappa_0^c - \mu_{wc}(\kappa_1^c - 1)) \tag{IA.12}$$
$$\lambda_x = -\gamma\phi_x - (\theta - 1)A(\kappa_1^c - \rho_x) \tag{IA.13}$$
$$\lambda_{\varepsilon x} = -(\theta - 1)A \tag{IA.14}$$
$$\lambda_{\varepsilon e} = \gamma\phi_e \tag{IA.15}$$
$$\lambda_{\varepsilon \mathcal{N}^c} = \gamma\phi_{\mathcal{N}^c} \tag{IA.16}$$
$$\lambda_{\varepsilon \mathcal{N}^s} = \gamma\phi_{\mathcal{N}^s}. \tag{IA.17}$$

The log SDF and the return on total wealth together yield

$$
\begin{aligned}
r^W_{t+1} + m_{t+1} &= \mu_s + r^c_0 + (\beta_x + \lambda_x)x_t \\
&\quad + (\beta_{\varepsilon e} - \lambda_{\varepsilon e})\sigma_e \varepsilon_{e,t+1} + (\beta_{\varepsilon x} - \lambda_{\varepsilon x})\sigma_x \varepsilon_{x,t+1} \\
&\quad + (\beta_{\varepsilon \mathcal{N}^C} - \lambda_{\varepsilon \mathcal{N}^C})\sigma_{\mathcal{N}^C} \varepsilon_{\mathcal{N}^C,t+1} + (\beta_{\varepsilon \mathcal{N}^S} - \lambda_{\varepsilon \mathcal{N}^S})\sigma_{\mathcal{N}^S} \varepsilon_{\mathcal{N}^S,t+1},
\end{aligned}
$$

and the Euler equation is given by

$$
\begin{aligned}
0 &= \mu_s + r^c_0 + (\beta_x + \lambda_x)\, x_t \\
&\quad + \frac{1}{2}\left[(\beta_{\varepsilon e} - \lambda_{\varepsilon e})^2 \sigma_e^2 + (\beta_{\varepsilon x} - \lambda_{\varepsilon x})^2 \sigma_x^2 + (\beta_{\varepsilon \mathcal{N}^C} - \lambda_{\varepsilon \mathcal{N}^C})^2 \sigma^2_{\mathcal{N}^C} + (\beta_{\varepsilon \mathcal{N}^S} - \lambda_{\varepsilon \mathcal{N}^S})^2 \sigma^2_{\mathcal{N}^S}\right].
\end{aligned}
$$

Using the method of undetermined coefficients, A solves

$$
0 = \beta_x + \lambda_x \implies A = \frac{\phi_x(1-\rho)}{\kappa^c_1 - \rho_x}. \tag{IA.18}
$$

Lastly, μ_{wc} solves

$$
\begin{aligned}
0 &= \mu_s + r^c_0 \\
&\quad + \frac{1}{2}\left[(\beta_{\varepsilon e} - \lambda_{\varepsilon e})^2 \sigma_e^2 + (\beta_{\varepsilon x} - \lambda_{\varepsilon x})^2 \sigma_x^2 + (\beta_{\varepsilon \mathcal{N}^C} - \lambda_{\varepsilon \mathcal{N}^C})^2 \sigma^2_{\mathcal{N}^C} + (\beta_{\varepsilon \mathcal{N}^S} - \lambda_{\varepsilon \mathcal{N}^S})^2 \sigma^2_{\mathcal{N}^S}\right].
\end{aligned}
$$

Therefore, we can use equation (IA.18) to calculate A. Furthermore, we can calculate prices of risk and betas of the return on total wealth by substituting A into equations (IA.6) to (IA.17).

E. Dividends

Let an individual stock's dividend growth process be described by

$$
\begin{aligned}
\Delta d_{i,t+1} &= \mu^i + \phi^i_x x_t + \varphi^i_e \sigma_e \varepsilon_{e,t+1} \\
&\quad + \varphi^i_{\mathcal{N}^C} \sigma_{\mathcal{N}^C} \varepsilon_{\mathcal{N}^C,t+1} + \varphi^i_{\mathcal{N}^S} \sigma_{\mathcal{N}^S} \varepsilon_{\mathcal{N}^S,t+1} + \zeta^i \sigma_i \varepsilon_{i,t+1}.
\end{aligned} \tag{IA.19}
$$

We guess and verify that the log price-dividend ratio is linear in the state variable of the model:

$$
pd_{i,t} = \mu^i_{pd} + A_i x_t. \tag{IA.20}
$$

Stock returns are approximated by

$$
r_{i,t+1} = \Delta d_{i,t+1} + \kappa^i_0 + \kappa^i_1 pd_{i,t+1} - pd_{i,t}, \tag{IA.21}
$$

where $\kappa^i_1 = \frac{\exp(\mu^i_{pd})}{1+\exp(\mu^i_{pd})}$ and $\kappa^i_0 = \log\left(1 + \exp\left(\mu^i_{pd}\right)\right) - \kappa^i_1 \mu^i_{pd}$ are the approximation constants. Next, we can derive the return on asset i by substituting equations (IA.19) and (IA.20) into the expression for the return on total wealth given by equation (IA.21):

$$
\begin{aligned}
r_{i,t+1} &= \Delta d_{i,t+1} + \kappa^i_0 + \kappa^i_1 pd_{i,t+1} - pd_{i,t} \\
&= r^i_0 + \beta^i_x x_t + \beta^i_{\varepsilon e} \sigma_e \varepsilon_{e,t+1} + \beta^i_{\varepsilon x} \sigma_x \varepsilon_{x,t+1} \\
&\quad + \beta^i_{\varepsilon \mathcal{N}^C} \sigma_{\mathcal{N}^C} \varepsilon_{\mathcal{N}^C,t+1} + \beta^i_{\varepsilon \mathcal{N}^S} \sigma_{\mathcal{N}^S} \varepsilon_{\mathcal{N}^S,t+1} + \beta^i_{\varepsilon i} \sigma_i \varepsilon_{i,t+1},
\end{aligned}
$$

where

$$
\begin{aligned}
r_0^i &= \mu^i + \kappa_0^i + \mu_{pd}^i(\kappa_1^i - 1) && \text{(IA.22)}\\
\beta_x^i &= \phi_x^i - A(1 - \kappa_1^i \rho_x) && \text{(IA.23)}\\
\beta_{\varepsilon x}^i &= \varphi_x^i + \kappa_1^i A^i && \text{(IA.24)}\\
\beta_{\varepsilon e}^i &= \varphi_e^i && \text{(IA.25)}\\
\beta_{\varepsilon \mathcal{N}^C}^i &= \varphi_{\mathcal{N}^C}^i && \text{(IA.26)}\\
\beta_{\varepsilon \mathcal{N}^S}^i &= \varphi_{\mathcal{N}^S}^i && \text{(IA.27)}\\
\beta_{\varepsilon i}^i &= \zeta^i. && \text{(IA.28)}
\end{aligned}
$$

The log SDF and the return on total wealth together yield

$$
\begin{aligned}
r_{t+1}^i + m_{t+1} &= \mu_s + r_0^i + (\beta_x^i + \lambda_x)x_t \\
&+ (\beta_{\varepsilon e}^i - \lambda_{\varepsilon e})\sigma_e \varepsilon_{e,t+1} + (\beta_{\varepsilon x}^i - \lambda_{\varepsilon x})\sigma_x \varepsilon_{x,t+1} \\
&+ (\beta_{\varepsilon \mathcal{N}^C}^i - \lambda_{\varepsilon \mathcal{N}^C})\sigma_{\mathcal{N}^C} \varepsilon_{\mathcal{N}^C,t+1} + (\beta_{\varepsilon \mathcal{N}^S}^i - \lambda_{\varepsilon \mathcal{N}^S})\sigma_{\mathcal{N}^S} \varepsilon_{\mathcal{N}^S,t+1}, + \beta_{\varepsilon i}^i \sigma_i \varepsilon_{i,t+1},
\end{aligned}
$$

and the Euler equation is given by

$$
\begin{aligned}
0 &= \mu_s + r_0^i + \left(\beta_x^i + \lambda_x\right) x_t + \frac{1}{2}\left[(\beta_{\varepsilon e}^i - \lambda_{\varepsilon e})^2 \sigma_e^2 + (\beta_{\varepsilon x}^i - \lambda_{\varepsilon x})^2 \sigma_x^2\right.\\
&\left. + (\beta_{\varepsilon \mathcal{N}^C}^i - \lambda_{\varepsilon \mathcal{N}^C})^2 \sigma_{\mathcal{N}^C}^2 + (\beta_{\varepsilon \mathcal{N}^S}^i - \lambda_{\varepsilon \mathcal{N}^S})^2 \sigma_{\mathcal{N}^S}^2 + (\beta_{\varepsilon i}^i)^2 \sigma_i^2\right].
\end{aligned}
$$

Using the method of undetermined coefficients, A^i solves

$$
0 = \beta_x^i + \lambda_x \implies A^i = \frac{\phi_x^i + \lambda_x}{1 - \kappa_1^i \rho_x}. \tag{IA.29}
$$

Lastly, μ_{pd}^i solves

$$
\begin{aligned}
0 &= \mu_s + r_0^i + \frac{1}{2}\left[(\beta_{\varepsilon e}^i - \lambda_{\varepsilon e})^2 \sigma_e^2 + (\beta_{\varepsilon x}^i - \lambda_{\varepsilon x})^2 \sigma_x^2 + (\beta_{\varepsilon \mathcal{N}^C}^i - \lambda_{\varepsilon \mathcal{N}^C})^2 \sigma_{\mathcal{N}^C}^2\right.\\
&\left. + (\beta_{\varepsilon \mathcal{N}^S}^i - \lambda_{\varepsilon \mathcal{N}^S})^2 \sigma_{\mathcal{N}^S}^2 + (\beta_{\varepsilon i}^i)^2 \sigma_i^2\right].
\end{aligned}
$$

Therefore, we can use equation (IA.29) to calculate A^i. Furthermore, we can calculate betas of the return on asset i by substituting A^i into equations (IA.22) to IA.28).

VII. Data Construction

The input-output network matrix is necessary to compute the asset pricing factors discussed in this paper. The main input-output data source is the BEA Input-Output Accounts, but this is only available on annual basis from 1997 to 2012. Due to the short sample, I compute an estimate of the input-output table based on Compustat segment customer data, which are available on annual basis from 1979 to 2013. If a customer represents more than 10% of the seller's sales, then the customer's name is reported in Compustat as well as the sales to that particular customer. Cohen and Frazzini (2008) located the customers' permanent number (PERMNO) from CRSP until 2009 and I updated (by hand) their data set locating the customer identification number up to 2013. Therefore, from this data set, it is possible to get entries of the network matrix W, but the resulting network is truncated since not all supplier-customer transactions are observed. The model is at the sector level, so to make data and model compatible, I aggregate Compustat customer sales data at the two-digit NAICS code level (sector level).

To mitigate the truncation issue, I consider three alternatives ways to compute the network matrix: (i) assume that all nonobserved entries are equal to zero and normalize each row to sum to one, (ii) equally distribute the remaining weight across sectors, or (iii) assume that all nonobserved entries are equal to zero and compute the factors based on the truncated network. In addition to the network calculation itself, we can compute the output shares in two ways as well: (i) use the Compustat reported total sales, or (ii) use only sales reported in the Compustat customer segment data in order to respect the network truncation. This results in two distinct ways to compute the shares δ. Hence, for each δ type, we can compute the concentration factor directly from δ, and the sparsity factor can be computed for each of the three alternative networks considered. To choose which method to use, I compare each of them to factors computed from the BEA input-output tables. The BEA data are from 1997 to 2012, so I choose the factor calculations based on their correlations with the BEA factors. The calculations that result in the highest correlations with the BEA data involve using Compustat customer segment sales and equally distributing the remaining network weights across sectors. Figure IA.3 reports the network factors' time series from Compustat and the factors from BEA Input-Output Accounts data. The network concentration factors from BEA and Compustat share a correlation of 86% in levels (6% in innovations), and the network sparsity series share a correlation of 54% in levels (24% in innovations).

Output shares also depend on both the network and the preference weights. The BEA input-output tables report the sector consumption by final consumer. Using the consumption of the final consumer to compute the preference weights, we can compare the concentration factor implied by the model (i.e., using the network and preference weights) with and without keeping the preference weights constant. Figure IA.4 reports the concentration factor using the BEA output shares directly (solid blue line), using model-implied shares keeping preference weights constant (average consumption expenditure shares), and using model-implied shares with consumption expenditure shares (α) varying over time as well. The resulting time series suggests that concentration is almost entirely driven by changes in the network rather than changes in the household's preferences.

VIII. Truncation analysis

Firms in Compustat do not have to report sales and customer names when sales represent less than 10% of their revenue. This data set has been used in recent academic research,[3] but the truncation is a limitation since we cannot uncover sales that have never been reported to Compustat. However, I show that my results are robust to additional truncation. Although we cannot observe unreported sales below the 10% threshold, we can artificially impose additional truncation to see how the results might change. This approach allows us to compare the empirical results based on artificially truncated data against the counterfactual of less truncation.

In this empirical exercise, I artificially increase the truncation from 10%, which generates the benchmark results and network factors, to 20%. For each truncation level considered, I reconstruct the network factors based on artificially truncated data, where I disregard sales below the artificial threshold for firms that report sales only above the 10% threshold. Figure IA.1 plots the time series of both network sparsity and network concentration for different truncation levels. Sparsity based on the original 10% truncated data is slightly different from the artificially truncated data around 1990 and after 2005, while the network concentration series are slightly different from each other around 2000 as well as earlier in the sample, during the 1980s.

[3] See Cohen and Frazzini (2008).

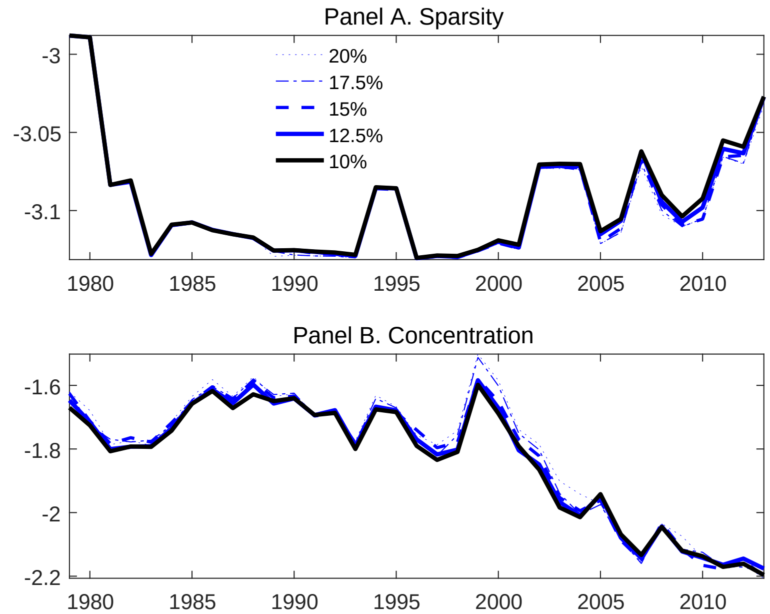

Figure IA.1. Artificial truncation: network factors. This figure plots the network factors for different truncation levels. In Panel A I plot network sparsity, and in Panel B I plot network concentration. I artificially increase the truncation from 10%, which generates the benchmark factors, to 20%. The data are truncated at 10%, which means that firms do not have to report sales that represent less than 10% of their revenue. Therefore, we do not observe small sales. I impose additional truncation by reconstructing the network factors based on artificially truncated Compustat sales data in which I disregard sales below the artificial truncation threshold from firms that report sales only above the 10% threshold.

In Table IA.III, I report the sparsity- and concentration-beta-sorted portfolio results in Panels A and B, respectively. In Columns (2) to (4), I report the average portfolio excess returns from low to high beta, and in Column (5) I report the average return on the long-short portfolio. The average return spread does not change much as the truncation increases. To test whether the truncation changes the return spread, I examine whether the difference between the average return on the long-short portfolio under artificially truncated data is different from the benchmark estimated return spread. In Column (6) I report this difference and in Column (7) I report the t-statistic. For all truncation levels considered, we cannot reject the null that the return deference is equal to zero. In fact, all t-statistics are below one in absolute value. This exercise suggests that my results do not depend heavily on data truncation per se, because the results are not statistically different from those based on artificially more truncated data.

Alternatively, we can compute the return spread based on BEA data, which are available at an annual frequency from 1997 to 2013. When I use the network factor based on the BEA data, the network-beta-sorted portfolio return spreads are consistent with the model but are not statistically significant. Sparsity and concentration long-short portfolios have average returns of 0.75% (t-statistic $= 0.22$) and -2.45% (t-statistic $= -0.78$) per year, respectively. To compute these return spreads, I estimated network betas on a rolling window of 11 years. This implies five years of returns to compute the return spreads, from January 2009 to December 2013. Note that, even with a very short sample, both sparsity- and concentration-beta-sorted portfolios have spread signs that are consistent with the theoretical model.

The BEA include benchmark input-output data for the years 1982, 1987, and 1992, as well.

Table IA.III
Artificial Truncation: Network Beta-Sorted Portfolios

This table reports the network-beta-sorted portfolio returns. Panel A reports the result for sparsity-beta-sorted portfolios, while Panel B reports the results for concentration-beta-sorted portfolios. In Column (1), I artificially increase the truncation from 10%, which generates the benchmark results, to 20%. The data are truncated at 10%, which means that firms do not have to report sales that represent less than 10% of their revenue. Therefore, we do not observe small sales. I impose additional truncation by reconstructing the network factors based on artificially truncated Compustat sales data, in which I disregard sales below the artificial truncation threshold from firms that report sales only above the 10% threshold. In Columns (2) to (4) I report the average portfolio excess returns, and in Column (5) I report the average return on a portfolio long the high-beta portfolio and short the low-beta portfolio. In Column (6) I report the average return difference between the long-short portfolio at the truncation specified in Column (1) and the benchmark results when the truncation is 10%. Finally, in Column (7) I report the t-statistic on the return difference reported in Column (6). The sample is from January 1995 to December 2013 at a monthly frequency.

Panel A: Sparsity-beta-sorted portfolios						
	Beta-sorted portfolio returns				Relative to benchmark	
Truncation	L		H	H-L	Difference	t-stat.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
10% (benchmark)	6.42	8.35	11.03	4.61	–	–
12.5%	6.18	8.68	10.52	4.35	−0.26	−0.34
15%	6.14	8.74	10.50	4.36	−0.25	−0.31
17.5%	6.31	8.89	10.65	4.34	−0.26	−0.29
20%	6.57	8.05	11.63	5.06	0.45	0.67
Panel B: Concentration-beta-sorted portfolios						
	Beta-sorted portfolio returns				Relative to benchmark	
Truncation	L		H	H-L	Difference	t-stat.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
10% (benchmark)	10.28	7.82	7.10	−3.18	–	–
12.5%	10.41	7.70	6.55	−3.87	−0.68	−0.73
15%	10.06	7.79	6.90	−3.16	0.02	0.02
17.5%	10.56	7.60	7.08	−3.48	−0.30	−0.29
20%	10.21	7.75	6.87	−3.34	−0.15	−0.19

The data for these years used the Standard Industry Classification (SIC), which is different from the North American Industry Classification System (Naics) used today. Nevertheless, we can match the two classification systems and have the input-output table at the two-digit Naics code. These additional data points have networks factors consistent with the factors from compustat. Furthermore, I found the results using these additional observations very similar to those using the most recent BEA sample (1997-2013): the network beta-sorted portfolio return spreads are consistent with the model, but not statistically significant.

Figure IA.2 plots standardized time series of each network factor from 1982 to 2013. In Panel A, I plot the sparsity factor from three data sets: Compustat, BEA (1997 to 2013), and historical BEA (1982 to 1992). In Panel B, I plot the concentration factor based on the same three data

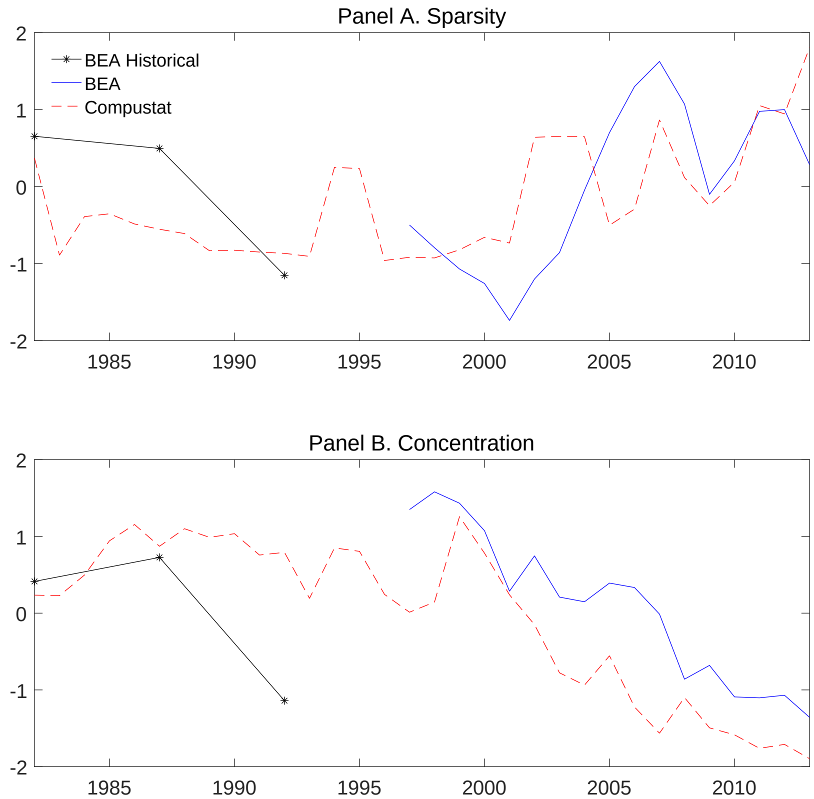

Figure IA.2. Network factors with historical BEA data. This figure plots standardized time series of the network factors from 1982 to 2013, from three different data sets: Compustat, BEA (1997 to 2013), and historical BEA (1982 to 1992). Panel A plots the sparsity factor time series, while Panel B plots the concentration factor time series.

sets. Although the historical BEA sample is not perfectly comparable to Compustat due to the change in industry classification, the earlier factors' time series based on the BEA data are still consistent with the factors based on Compustat: sparsity declines in the two quinquennial changes (from 1982 to 1987 and from 1987 to 1992) in both samples, and concentration increases in the first quinquennial change (from 1982 to 1987) in both samples.

We can also run the portfolio sorting exercise for the BEA factors using these three additional observations. Since the first three observations are at a five-year frequency, I use quinquennial returns in the beginning of sample to build network betas. Sparsity and concentration long-short portfolios have the correct sign on the spreads, with average returns of 0.93% (t-statistic $= 0.27$) and -0.09% (t-statistic $= -0.03$), respectively. Similar to the previous analysis, I estimated network betas on a rolling window of 11 observations to compute these return spreads. Again, not that even with a very short sample, both sparsity- and concentration-beta-sorted portfolios have spread signs consistent with the theoretical model.

IX. Additional Figures and Tables

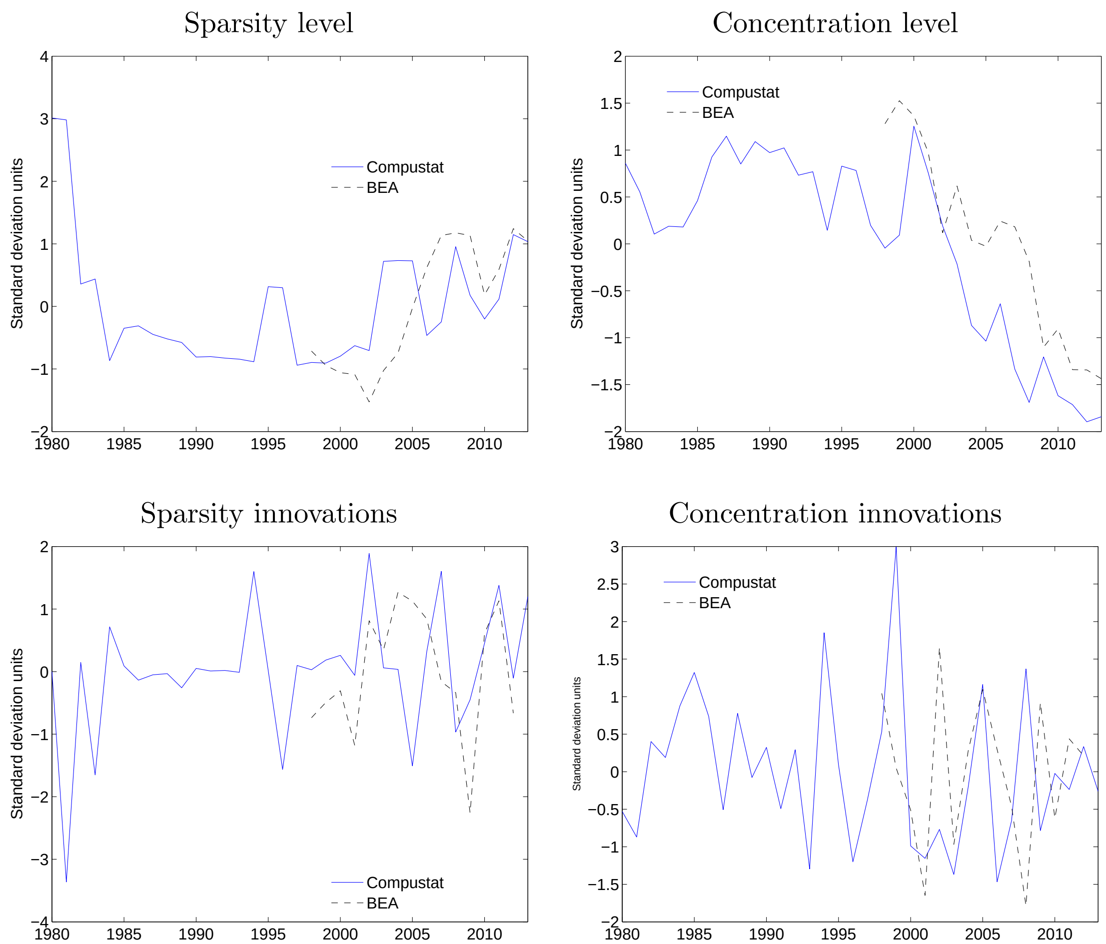

Figure IA.3. BEA amd Compustat network factors: using total customer segment sales. This figure plots sparsity and concentration factors computed from Compustat (solid line) and from the Bureau of Economic Analysis (BEA) Input-Output Accounts (dashed line). Standardized series are plotted in levels and innovations (one period change).

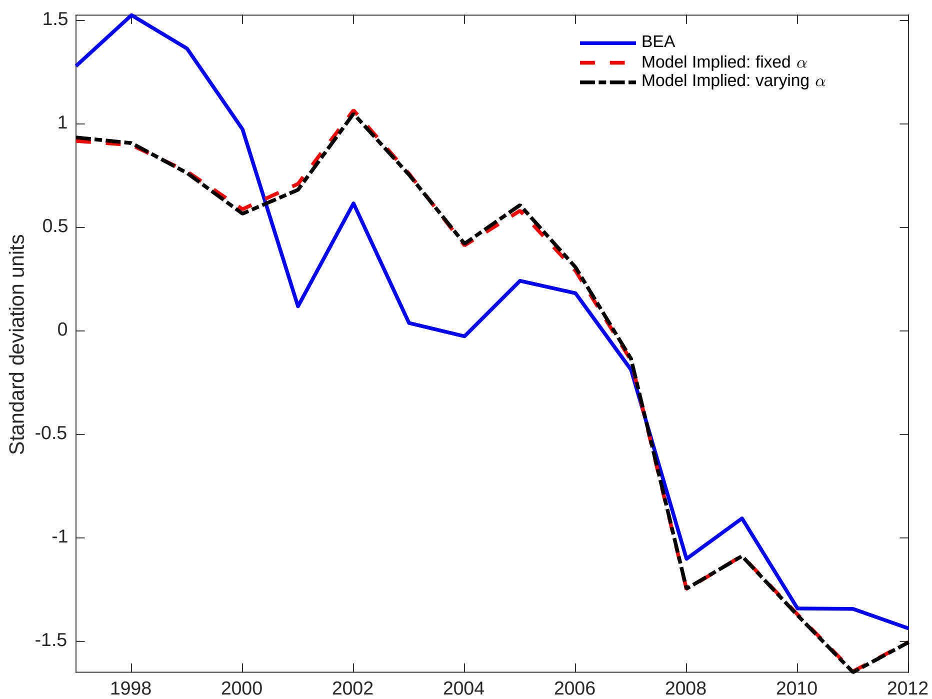

Figure IA.4. Concentration factors: model-implied shares versus shares from BEA. This figure plots the concentration factor using the BEA output shares directly (solid blue line), using model-implied shares keeping alpha constant (average consumption expenditure shares), and using model-implied shares with consumption expenditure shares (alphas) varying over time as well. All series are standardized.

Table IA.IV
Double Sort

This table reports average excess returns in Panel A, post-sample CAPM alphas in Panel B, and post-sample Fama and French (1993) alphas in Panel C. Stocks are independently double-sorted on sparsity beta and concentration beta, and double-sorted portfolios are formed by terciles.

Panel A: Returns					
	Concentration				
Sparsity	(1)	(2)	(3)	(3)-(1)	t-stat.
(1)	11.12	7.43	6.30	−4.82	−1.40
(2)	11.65	6.79	8.74	−2.91	−1.27
(3)	10.95	10.78	14.62	3.67	0.97
(3)-(1)	−0.17	3.36	8.32	–	–
t-stat.	−0.04	1.05	2.33	–	–
Panel B: α_{CAPM}					
	Concentration				
Sparsity	(1)	(2)	(3)	(3)-(1)	t-stat.
(1)	1.98	0.02	−1.73	−3.71	−1.08
(2)	4.27	1.15	2.48	−1.79	−0.79
(3)	4.16	4.80	7.83	3.67	0.95
(3)-(1)	2.18	4.78	9.56	–	–
t-stat.	0.54	1.50	2.68	–	–
Panel C: α_{FF}					
	Concentration				
Sparsity	(1)	(2)	(3)	(3)-(1)	t-stat.
(1)	1.15	−0.42	−2.28	−3.43	−1.05
(2)	3.92	0.09	1.30	−2.62	−1.19
(3)	3.03	3.54	8.26	5.23	1.46
(3)-(1)	1.88	3.96	10.54	–	–
t-stat.	0.50	1.24	3.05	–	–

Table IA.V
Robustness

This table reports average excess returns in Panel A, post-sample CAPM alphas in Panel B, and post-sample Fama and French (1993) alphas in Panel C. Stocks are sorted on sparsity beta and concentration beta, and one-way-sorted portfolios are formed by terciles. Each row corresponds to a different specification: (1) benchmark estimation, (2) excludes network factors in levels from the exposure estimation in equation (24), (3) excludes residual TFP from the exposure estimation, (4) excludes network factors in levels and residual TFP from the exposure estimation, (5) uses innovations in residual TFP as consumption growth orthogonalized to the network factors' innovations, (6) uses a 16-year trailing window, (7) uses a 17-year trailing window, (8) uses a 18-year trailing window, (9) uses a 19-year trailing window, and (10) uses a 20-year trailing window.

		Sparsity-beta-sorted portfolios					Concentration-beta-sorted portfolios				
		(1)	(2)	(3)	(3)-(1)	*t*-stat.	(1)	(2)	(3)	(3)-(1)	*t*-stat.
Panel A: Average returns											
1.	Benchmark	6.42	8.35	11.03	4.61	2.08	10.28	7.82	7.10	−3.18	−2.02
2.	No level control	6.19	8.67	10.43	4.24	2.04	9.46	8.97	6.47	−2.99	−1.55
3.	No Res. TFP control	6.41	8.87	10.41	4.00	1.69	10.68	7.94	6.80	−3.88	−2.32
4.	No level and Res. TFP	6.70	8.64	10.48	3.78	1.91	10.98	8.46	6.69	−4.28	−2.11
5.	R. TFP from Cons.	6.39	9.04	10.91	4.51	1.88	10.05	7.83	7.03	−3.02	−1.77
6.	16-year window	5.16	7.37	10.41	5.25	2.07	10.07	6.56	5.80	−4.27	−2.57
7.	17-year window	4.68	6.91	9.09	4.42	1.58	8.86	6.59	4.46	−4.40	−2.09
8.	18-year window	3.13	6.02	7.76	4.63	1.57	7.23	4.85	3.69	−3.54	−1.71
9.	19-year window	0.85	5.29	7.83	6.98	2.28	6.29	4.35	2.45	−3.84	−1.84
10.	20-year window	2.07	5.21	7.00	4.93	1.83	5.68	4.41	3.21	−2.47	−1.24
Panel B: CAPM alphas											
		Sparsity-beta-sorted portfolios					Concentration-beta-sorted portfolios				
		(1)	(2)	(3)	(3)-(1)	*t*-stat.	(1)	(2)	(3)	(3)-(1)	*t*-stat.
1.	Benchmark	−1.50	2.25	4.68	6.18	2.91	2.98	1.85	−0.31	−3.30	−2.07
2.	No level control	−1.98	2.65	4.07	6.05	3.14	2.40	2.98	−0.95	−3.34	−1.72
3.	No Res. TFP control	−1.72	2.95	3.88	5.59	2.44	3.06	1.95	−0.51	−3.57	−2.12
4.	No level and Res. TFP	−1.45	2.59	4.24	5.69	3.17	3.76	2.48	−0.59	−4.35	−2.11
5.	R. TFP from Cons.	−1.91	3.10	4.42	6.33	2.78	2.75	1.78	−0.41	−3.16	−1.83
6.	16-year window	−1.76	2.04	5.08	6.84	2.84	3.62	1.59	−0.62	−4.23	−2.53
7.	17-year window	−1.94	1.85	4.28	6.21	2.40	2.97	1.75	−1.59	−4.56	−2.16
8.	18-year window	−2.54	1.85	3.81	6.35	2.38	2.26	0.85	−1.33	−3.59	−1.72
9.	19-year window	−3.92	1.94	4.65	8.57	3.17	2.29	0.97	−1.63	−3.92	−1.87
10.	20-year window	−1.61	2.65	4.38	5.99	2.54	2.44	1.85	−0.02	−2.45	−1.23
Panel C: Fama and French alphas											
		Sparsity-beta-sorted portfolios					Concentration-beta-sorted portfolios				
		(1)	(2)	(3)	(3)-(1)	*t*-stat.	(1)	(2)	(3)	(3)-(1)	*t*-stat.
1.	Benchmark	−1.82	1.27	3.66	5.49	2.65	1.97	0.95	−0.76	−2.74	−1.72
2.	No level control	−2.57	1.75	3.04	5.61	2.95	1.00	1.89	−1.24	−2.23	−1.20
3.	No Res. TFP control	−1.78	1.81	2.72	4.49	2.03	2.19	1.04	−1.01	−3.20	−1.89
4.	No level and Res. TFP	−2.04	1.51	3.42	5.46	3.01	1.95	1.33	−0.79	−2.74	−1.44
5.	R. TFP from Cons.	−2.01	2.06	3.20	5.21	2.35	1.89	1.02	−0.97	−2.86	−1.64
6.	16-year window	−1.99	1.19	4.06	6.04	2.74	2.83	0.66	−1.09	−3.92	−2.34
7.	17-year window	−2.16	1.19	3.37	5.53	2.23	2.04	1.17	−2.16	−4.20	−2.01
8.	18-year window	−2.60	1.60	3.11	5.71	2.23	1.57	0.45	−1.67	−3.24	−1.57
9.	19-year window	−3.57	2.10	4.07	7.63	2.92	1.47	0.52	−1.31	−2.77	−1.37
10.	20-year window	−1.32	2.13	3.52	4.84	2.04	0.63	0.72	0.52	−0.11	−0.06

Table IA.VI
Average β Correlations

This table reports the time-series average of the cross-sectional correlation between the β estimates for all 10 specifications of Table IA.V

		Correlations		
		$(\beta_{\mathcal{N}^S,t}, \beta_{\mathcal{N}^C,t})$	$(\beta_{\mathcal{N}^S,t}, \beta_{e,t})$	$(\beta_{\mathcal{N}^C,t}, \beta_{e,t})$
1.	Benchmark	−0.09	0.21	−0.18
2.	No level control	−0.18	0.39	−0.06
3.	No Res. TFP control	−0.08	–	–
4.	No level and Res. TFP	−0.18	–	–
5.	R. TFP from Cons.	−0.07	0.16	0.18
6.	16-year window	−0.08	0.18	−0.15
7.	17-year window	−0.07	0.15	−0.11
8.	18-year window	−0.04	0.11	−0.07
9.	19-year window	−0.02	0.08	−0.03
10.	20-year window	0.01	0.05	−0.00

Table IA.VII
Double Sorts on Sparsity and Other Factors

I construct double-sorted portfolios using stocks sorted on sparsity beta and on another factor. This table reports average excess returns, post-sample CAPM alphas and post-sample Fama and French (1993) alphas for the portfolio long on the high-sparsity beta and short on the low-sparsity beta. I consider seven different factors for the double sort: (1) market value, (2) book-to-market ratio, (3) total volatility, (4) idiosyncratic volatility from the CAPM model (standard deviation over one year of daily data), (5) idiosyncratic volatility from the Fama and French (1993) three-factor model (standard deviation over one year of daily data), (6) volume, and (7) turnover.

			Returns		α_{CAPM}		α_{FF}	
			(3)-(1)	t-stat.	(3)-(1)	t-stat.	(3)-(1)	t-stat.
1.	Market Value	L	2.71	1.56	2.52	1.44	2.62	1.48
		H	4.60	2.07	6.18	2.91	5.50	2.64
2.	Book to Market	L	4.59	2.01	5.93	2.66	5.10	2.35
		H	2.16	0.67	4.44	1.44	4.15	1.34
3.	Total Vol.	L	4.74	2.20	6.11	2.92	5.31	2.60
		H	4.99	1.12	5.07	1.13	5.93	1.31
4.	Idiosyncratic Vol. (CAPM)	L	4.81	2.17	6.33	2.96	5.60	2.68
		H	1.66	0.36	2.73	0.59	3.78	0.81
5.	Idiosyncratic Vol. (FF)	L	4.93	2.21	6.50	3.04	5.79	2.76
		H	−0.65	−0.14	0.15	0.03	0.81	0.17
6.	Volume	L	0.52	0.32	0.33	0.21	0.55	0.34
		H	4.59	2.06	6.19	2.90	5.50	2.63
7.	Turnover	L	4.10	1.75	4.30	1.81	2.93	1.29
		H	4.98	1.77	6.76	2.48	6.27	2.30

Table IA.VIII
Double Sorts on Concentration and Other Factors

I construct double-sorted portfolios using stocks sorted on concentration beta and on another factor. This table reports average excess returns, post-sample CAPM alphas and post-sample Fama and French (1993) alphas for the portfolio long on the high-sparsity beta and short on the low-sparsity beta. I consider seven different factors for the double sort: (1) market value, (2) book-to-market ratio, (3) total volatility, (4) idiosyncratic volatility from the CAPM model (standard deviation over one year of daily data), (5) idiosyncratic volatility from the Fama and French (1993) three-factor model (standard deviation over one year of daily data), (6) volume, and (7) turnover.

			Returns		α_{CAPM}		α_{FF}	
			(3)-(1)	*t*-stat.	(3)-(1)	*t*-stat.	(3)-(1)	*t*-stat.
1.	Market Value	L	−1.78	−0.96	−2.27	−1.21	−1.93	−1.03
		H	−3.16	−2.00	−3.28	−2.05	−2.73	−1.71
2.	Book to Market	L	−3.12	−1.78	−2.97	−1.67	−2.35	−1.32
		H	−2.40	−0.98	−4.04	−1.72	−4.31	−1.81
3.	Total Vol.	L	−3.10	−2.03	−3.23	−2.08	−2.79	−1.79
		H	−6.21	−1.32	−5.99	−1.25	−5.36	−1.11
4.	Idiosyncratic Vol. (CAPM)	L	−2.92	−1.87	−3.07	−1.94	−2.55	−1.62
		H	−5.42	−1.13	−5.58	−1.15	−5.04	−1.03
5.	Idiosyncratic Vol. (FF)	L	−3.06	−1.95	−3.27	−2.06	−2.78	−1.75
		H	−3.11	−0.65	−2.93	−0.61	−1.95	−0.40
6.	Volume	L	−1.06	−0.54	−2.05	−1.06	−2.04	−1.04
		H	−3.13	−1.98	−3.24	−2.02	−2.69	−1.69
7.	Turnover	L	−3.59	−1.82	−3.36	−1.68	−2.46	−1.25
		H	−3.89	−1.71	−4.33	−1.89	−3.95	−1.73

Table IA.IX

Fama and MacBeth Analysis: Controling for Fama and French Factors

This table reports the estimated prices of risk for three asset pricing factors: sparsity and concentration network factor-mimicking portfolios, along with the residual TFP factor-mimicking portfolio. In the estimation, I also control for the Fama and French (1993) factors, namely, the market excess return, the small minus big, and the high minus low portfolios. In terms of test assets, I consider seven different test assets. Column (1) uses 25 portfolios double-sorted on size and book-to-market. Column (2) adds 10 momentum-sorted portfolios. Column (3) adds 10 long- and 10 short-term reversal sorted portfolios. Column (4) adds 10 investment and 10 operating profitability sorted portfolios. Column (5) adds 10 accruals, 10 cash flow, 10 dividend yield, 10 earning-price ratio, 10 net issuance, 10 residual variance, and 10 total variance sorted portfolios. Column (6) adds four corporate bond portfolios sorted by credit rating. Different from the previous columns, Column (7) uses only 30 industry-sorted portfolios. All the portfolio data come from Kenneth French's website, except for the corporate bond portfolios, which are from Citibank's Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. The last row reports the cross-sectional R^2. t-statistics are based on Newey and West (1987) standard errors. The sample period is from January 1995 to December 2013 at a monthly frequency (228 months), except for the corporate bond portfolio sample, which ends on May 2013. All coefficients are annualized.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Residual TFP	2.25	0.62	8.76	9.12	4.15	5.48	3.73
t-stat.	0.41	0.14	2.70	3.83	3.06	3.84	1.97
Sparsity	−1.61	−2.65	1.30	3.91	4.31	6.35	5.97
t-stat.	−0.30	−0.78	0.41	2.31	2.71	3.69	3.41
Concentration	−5.48	−7.85	−8.30	−6.98	−5.67	−5.36	−4.98
t-stat.	−2.02	−2.63	−3.90	−4.82	−4.48	−4.10	−2.06
R^2	0.66	0.72	0.56	0.57	0.36	0.37	0.41

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